Intermediate Microeconomics (22014)

I. Consumer Theory Applications

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First Semester, 2011

Outline Part I. Consumer Theory Applications

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   - 1.2 Preferences
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TOPIC 0. CONSUMER THEORY REVIEW

Budget Constraints

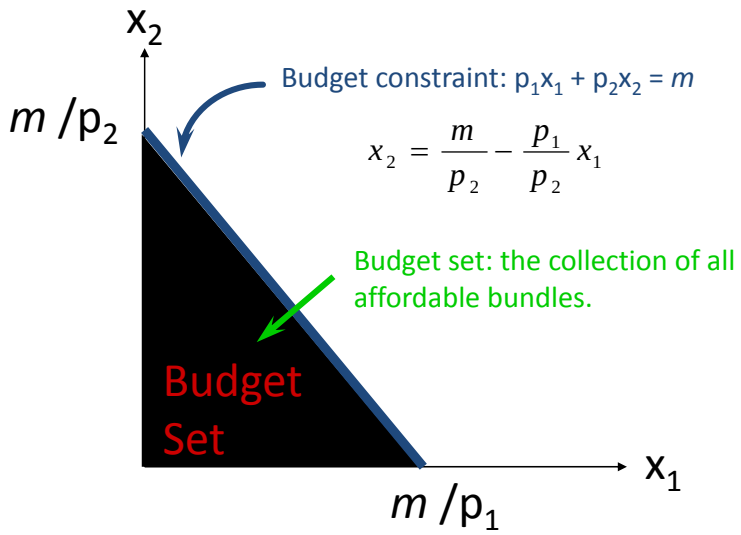
Definitions

The consumer's ***budget set*** is the set of all affordable bundles,

$$B(p_1, .., p_n; m) =$$

$$\{(x_1, \ldots, x_n) : x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1 x_1 + \ldots + p_n x_n \leq m\}.$$

The ***budget constraint*** is the upper boundary of the budget set.

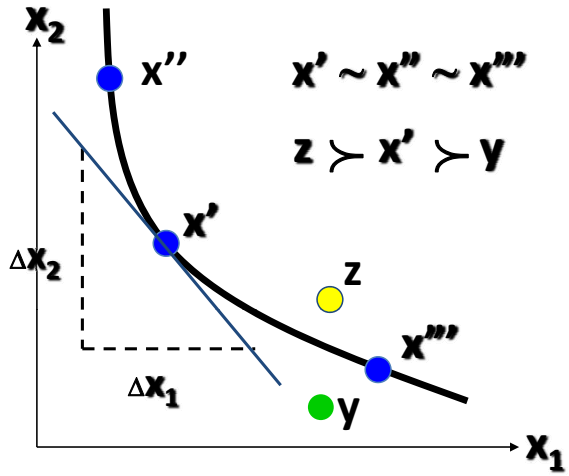


Preferences

Definitions

The set of all bundles equally preferred to a bundle x' is the ***indifference curve*** containing x'. The slope of the indifference curve at x' is the ***marginal rate of substitution*** (MRS) at x', which is the rate at which the consumer is only just willing to exchange commodity 2 for commodity 1':

$$MRS(x') = \lim_{\triangle x_1 \to 0} \left(\frac{\triangle x_2}{\triangle x_1} \right) = \frac{dx_2}{dx_1}$$



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Utility Function

Definition

A ***Utility function*** $U(x)$ represents a preference relation if and only if

$$x' \succ x'' \leftrightarrow U(x') > U(x'')$$
$$x' \prec x'' \leftrightarrow U(x') < U(x'')$$
$$x' \sim x'' \leftrightarrow U(x') = U(x'')$$

- The general equation for an indifference curve is $U(x_1, x_2) = k$, where k is a constant. Totally differentiating this identity we obtain that the MRS is equal to the ratio of marginal utilities:

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0 \Leftrightarrow \frac{dx_2}{dx_1} = -\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}}$$

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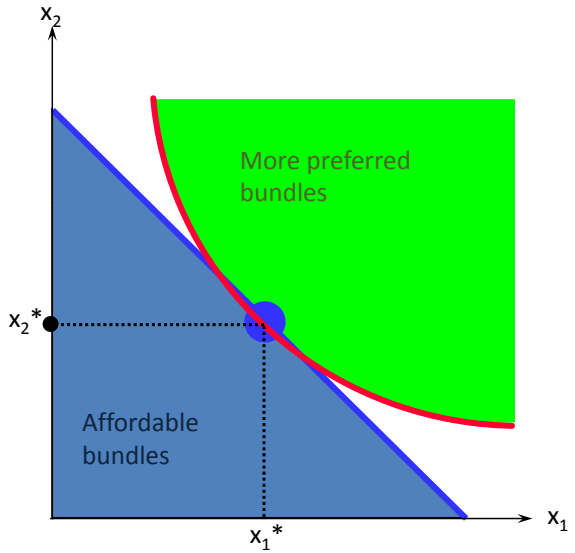
Choice

Definition

A decisionmaker chooses the most preferred affordable bundle, which is called the consumer's ***ordinary demand*** or gross demand.

- The slope of the indifference curve at ordinary demand (x_1^*, x_2^*) equals the slope of the budget constraint:

$$MRS(x_1^*, x_2^*) = -\frac{p_1}{p_2} \Leftrightarrow \frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{p_1}{p_2}$$



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Slutsky Equation

Changes to demand from a ***price change*** are always the sum of a ***pure substitution effect*** and an ***income effect***.

- ***Pure substitution effect***: change in demand due only to the change in relative prices. "What is the change in demand when the consumer's income is adjusted so that, at the new prices, she can only just buy the original bundle?"

- ***Income effect:*** if, at the new prices, less income is needed to buy the original bundle then "real income" is increased; if more income is needed, then "real income" is decreased.

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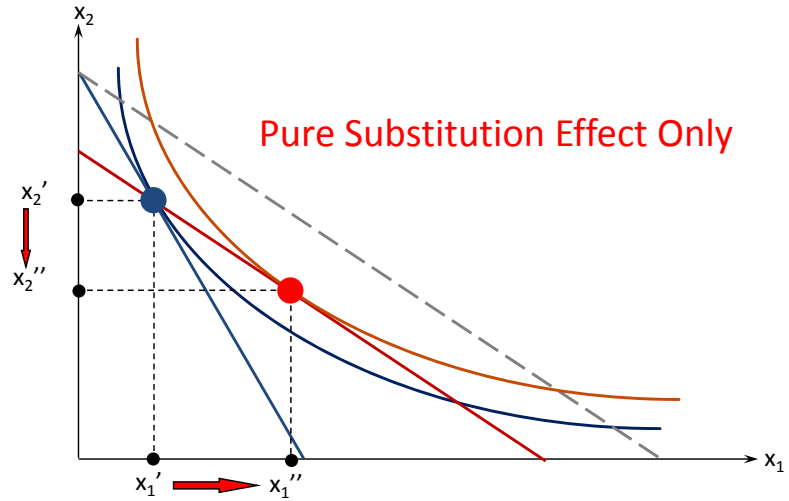
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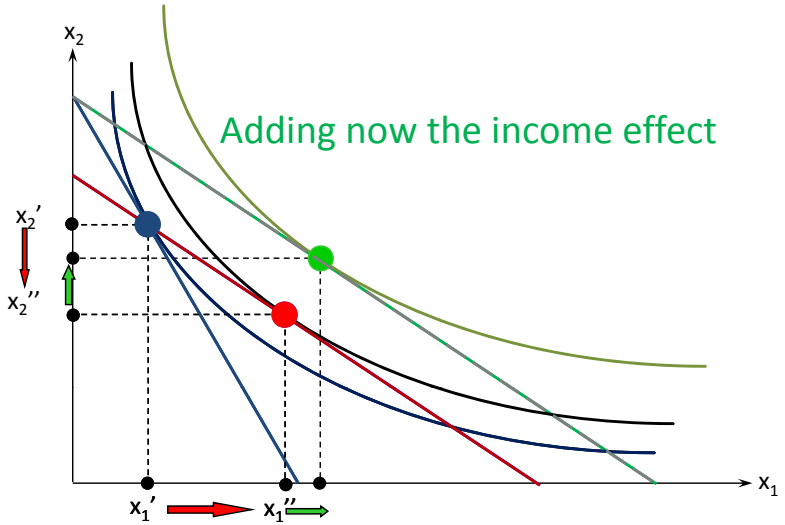
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Slutsky Equation graphically





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Slutsky Equation formally

- Let (p_1, p_2) be the initial price vector, m the income level and (x_1^*, x_2^*) the initial gross demand.
- Define the Sltusky demand function x_1^s as the demand of good i after the price change when income is adjusted to give the consumer just enough to consume the initial bundle (x_1^*, x_2^*):

$$x_i^s\left(p_1', p_2', x_1^*, x_2^*\right) \equiv x_i\left(p_1', p_2', \underbrace{p_1' x_1^* + p_2' x_2^*}_{m}\right)$$

- Take the derivative with respect to p_1 on both sides,

$$\frac{\partial x_1^s}{\partial p_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m} x_1^*$$

$$\Leftrightarrow$$

$$\frac{\partial x_1}{\partial p_1} = \underbrace{\frac{\partial x_1^s}{\partial p_1}}_{\text{substitution effect}} \underbrace{- \frac{\partial x_1}{\partial m} x_1^*}_{\text{income effect}}$$

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Slutsky Equation formally

- The sign of the susbstitution effect $\frac{\partial x_1^s}{\partial p_1}$ is negative: the cange in demand due to the susbstitution effect is the opposite to the change in price ($p_1 \uparrow \Rightarrow x_1^s \downarrow$ and $p_1 \downarrow \Rightarrow x_1^s \uparrow$).
- If the good is normal, the sign of the income effect is also negative: an increase in a price is like a decrease in income, which leads to a decrease in demand; a price fall is like an income increase, which leads to an increase in demand.

$$\frac{\partial x_1}{\partial p_1} = \underbrace{\frac{\partial x_1^s}{\partial p_1}}_{(-)} \underbrace{- \underbrace{\frac{\partial x_1}{\partial m}}_{(+)} x_1^*}_{(-)}$$

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TOPIC 1. BUYING AND SELLING

- So far, consumers' income taken as exogenous and independent of prices. In reality, consumers' income coming from exchange by sellers and buyers.
- How are incomes generated? How does the value of income depend upon commodity prices?
- How can we put all this together to explain better how price changes affect demands?

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Endowments

- In this chapter, consumers get income from endowments. This makes the budget set definition change slightly.

Definition

The list of resource units with which a consumer starts is her ***endowment***, denoted by $\omega = (\omega_1, \omega_2)$.

Definitions

Given p_1 and p_2, the ***budget constraint*** for a consumer with an endowment $\omega = (\omega_1, \omega_2)$ is

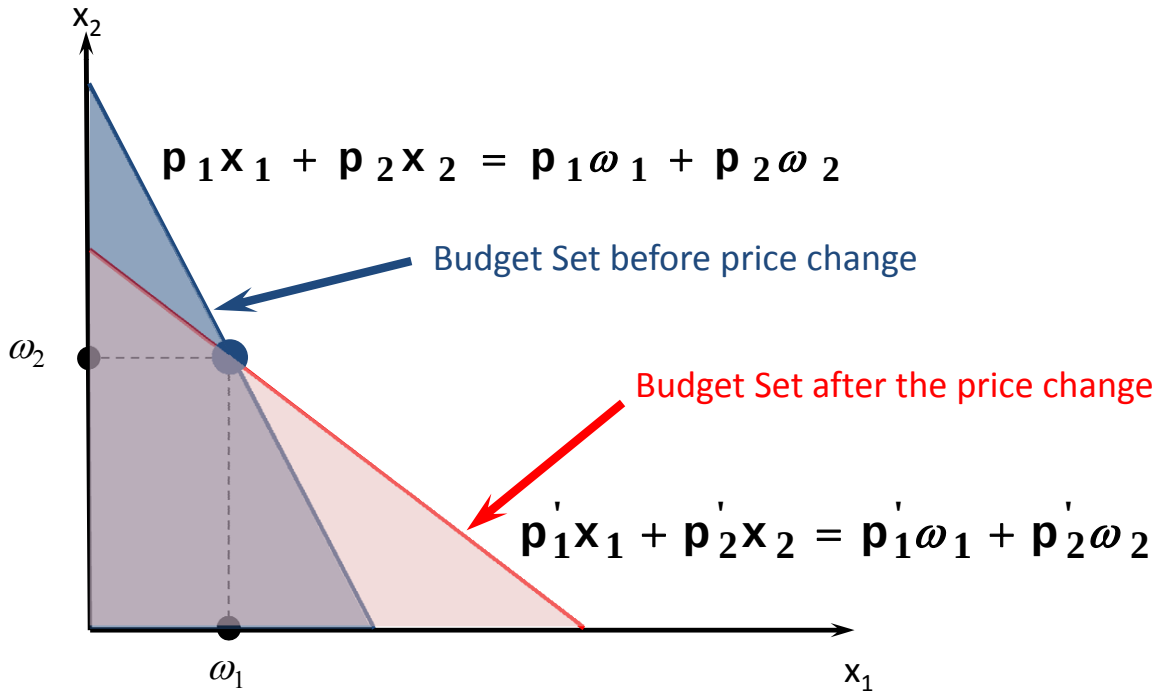
$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2.$$

and the ***budget set*** is formally defined as

$$\{(x_1, x_2) \mid x_1 \geq 0, x_2 \geq 0 \text{ and } p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2\}.$$

Endowments and Budget Sets

- Graphically, the endowment point is always on the budget constraint.
- Hence, price changes pivot the constraint around the endowment point.



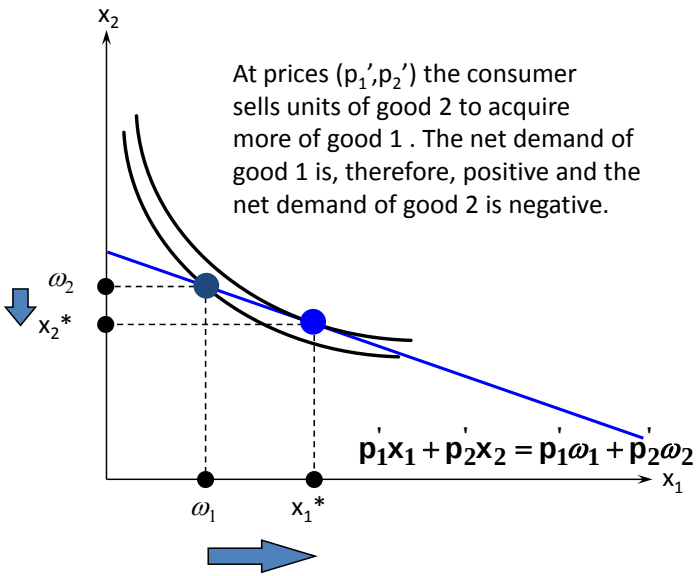
Net demand

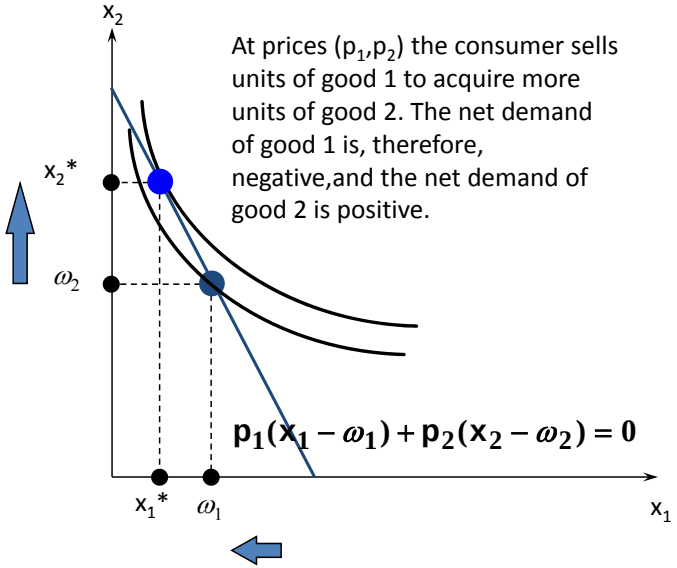
Definition

The difference between final consumption and initial endowment of a given good i, $x_i^* - \omega_i$, is called ***net demand of good*** i.

- The sum of the values of net demands is zero:

$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$$
$$\Leftrightarrow p_1 (x_1 - \omega_1) + p_2 (x_2 - \omega_2) = 0.$$





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Price offer curve

Definition

Price-offer curve contains all the utility-maximizing gross demands for which the endowment is exchanged.

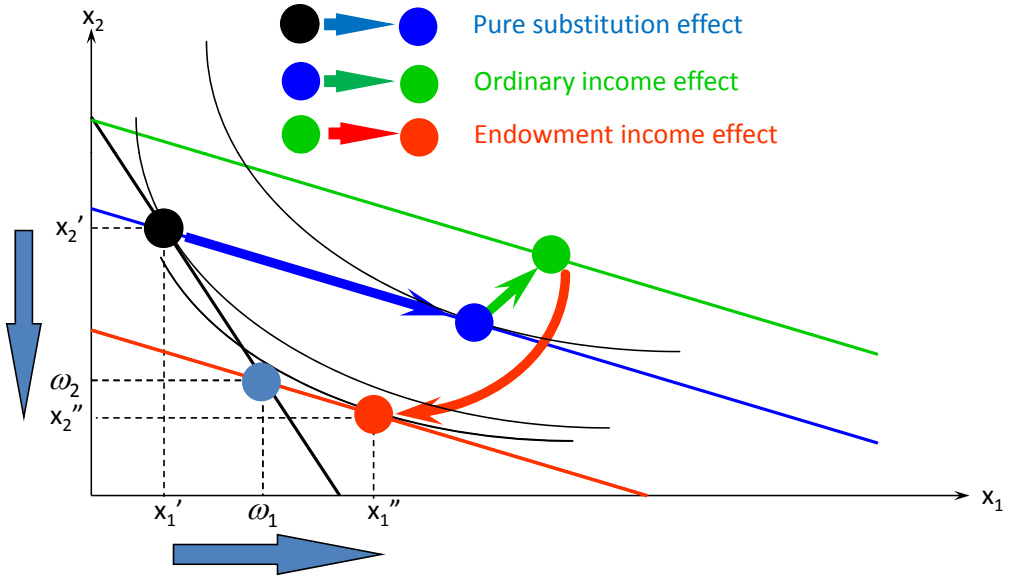


Slutsky equation revisited

In an endowment economy, the overall change in demand caused by a price change is the sum of *a pure substitution effect*, *an (ordinary) income effect*, ***and an endowment income effect***.

- *Pure Substitution Effect*: effect of relative prices change.
- *Income Effect*: effect of original bundle cost change.
- ***Endowment Income Effect:*** change in demand due only to the change in endowment value.

Price change from (p_1',p_2') to (p_1'', p_2'):



Slutsky Equation revisited

- Let $x_1(p_1, p_2; m(p_1, p_2))$ be the demand function of good 1 and $m(p_1, p_2) = p_1\omega_1 + p_2\omega_2$ the money income.
- Then, the total derivative of x_1 with respect to p_1 is

$$\frac{dx_1}{dp_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m}\omega_1.$$

- Using that $\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^s}{\partial p_1} - \frac{\partial x_1}{\partial m}x_1^*$, we obtain

$$\frac{dx_1}{dp_1} = \underbrace{\frac{\partial x_1^s}{\partial p_1}}_{\text{substitution}} \underbrace{-\frac{\partial x_1}{\partial m}x_1^*}_{\text{ord.-income}} \underbrace{+\frac{\partial x_1}{\partial m}\omega_1}_{\text{end.-income}}.$$

- Rearranging,

$$\frac{dx_1}{dp_1} = \underbrace{\frac{\partial x_1}{\partial p_1}}_{(-)} + (\omega_1 - x_1)\underbrace{\frac{\partial x_1}{\partial m}}_{(+)}.$$

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Slutsky equation revisited

Overall change in demand of normal good (demand increases with income) caused by own price change:

- When income is exogenous, both the substitution and (ordinary) income effects increase demand after an own-price fall; hence, a normal good's ordinary demand curve slopes down (thus, Law of Downward-Sloping Demand always applies to normal goods when income is exogenous).

- When income is given by initial endowments, endowment-income effect decreases demand if consumer supplies that good (negative net demand); thus, if the endowment income effect offsets the substitution and the (ordinary) income effects, the demand function could be upward-sloping!

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An application: labor supply

Environment description:

- A worker is endowed with m euros of nonlabor income and $\overline{R}$ hours of time.
- Consumption good's price is p_c, and the wage rate is w.
- Worker decides amount of consumption good, denoted by C, and amount of leisure, denoted by R.

Budget constraint:

$$p_c C = m + w\left(\overline{R} - R\right)$$

$$\Leftrightarrow \quad \underbrace{p_c C + wR}_{\text{Expenditures value}} = \underbrace{m + w\overline{R}}_{\text{Endowment value}}$$

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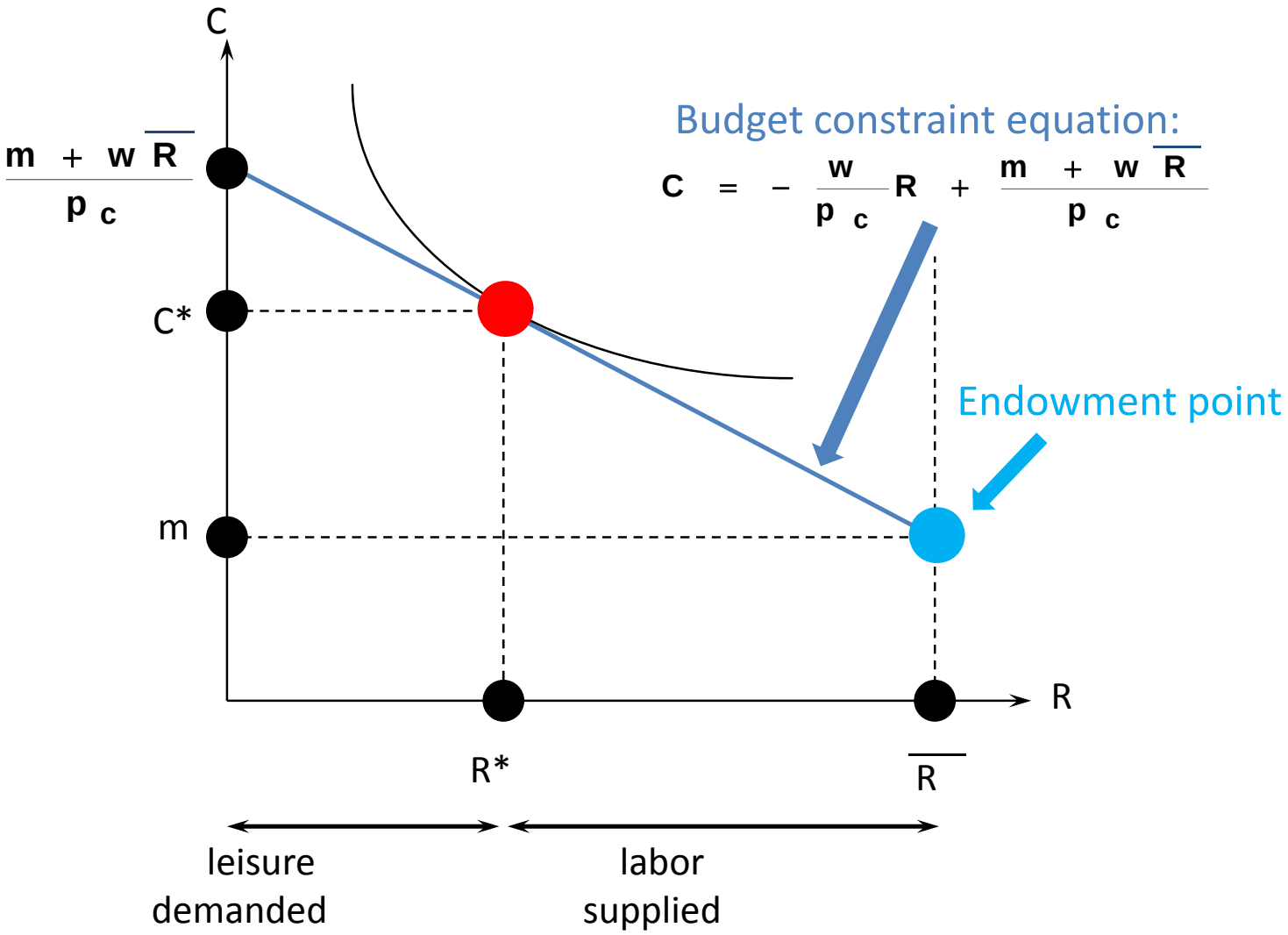
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Labor supply choice



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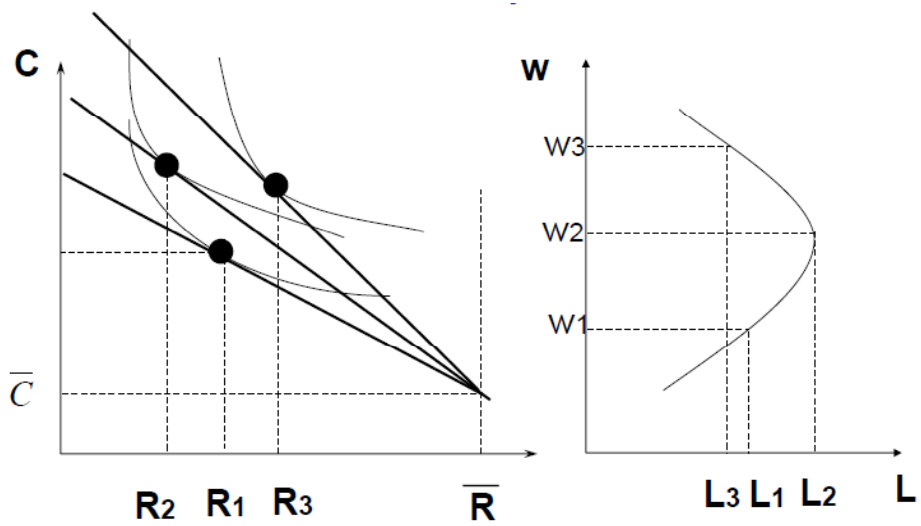
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Labor supply curve

Effect of a wage rate increase on amount labor supplied:

- Substitution effect: leisure relatively more expensive $\rightarrow$ decrease leisure demanded / increase labor supplied.
- (Ordinary) income effect: cost original bundle increases$\rightarrow$ decrease leisure demanded / increase labor supplied.
- Endowment-income effect: positive endowment income effect because worker supplies labor $\rightarrow$ decrease leisure demanded / increase labor supplied.

$\Rightarrow$ Labor supply curve may bend backwards.



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TOPIC 2. INTERTEMPORAL CHOICE

- So far, only static problems considered, as if consumers only alive one period or only static decisions.
- However, in the real world people often make intertemporal consumption decisions:
  - Current consumption financed by borrowing now against income to be received in the future.
  - Extra income received now spread over the following month (saving now for consumption later).
- In this section, we study intertemporal choice problem using a two-period version of our consumer's choice model.

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Intertemporal Choice Problem

- Notation:
  - Let interest rate be denoted by r.
  - Let c_1 and c_2 be consumptions in periods 1 and 2.
  - Let m_1 and m_2 be incomes received in periods 1 and 2. .
  - Let consumption prices be denoted by p_1 and p_2.
- Intertemporal choice problem:
  - ***Given incomes*** m_1 ***and*** m_2***, and given consumption prices*** p_1 ***and*** p_2***, what is the most preferred intertemporal consumption bundle*** (c_1, c_2)***?***
  - Need to know: the intertemporal budget constraint, and intertemporal consumption preferences.

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Present and Future Values

Definitions

Given an interest rate r, the ***future value*** of $M€$ is the value next period of that amount saved now:

$$FV = M(1+r).$$

The ***present value*** of $M€$ is the amount saved in the present to obtain $M€$ at the start of the next period:

$$PV = \frac{M}{1+r}.$$

- Example:
  - Example: if r $= 0.1$ the future value of $100€$ is $100(1+0.1) = 110€$.
  - if r=0.1, the present value of $1€$ is the amount we have to pay now to obtain $1€$ next period: $\frac{1}{1+0.1} = 0.91$.

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Intertemporal Budget Constraint

Case I: No inflation , $p_1 = p_2$

- Consumption bundle when neither saving nor borrowing:

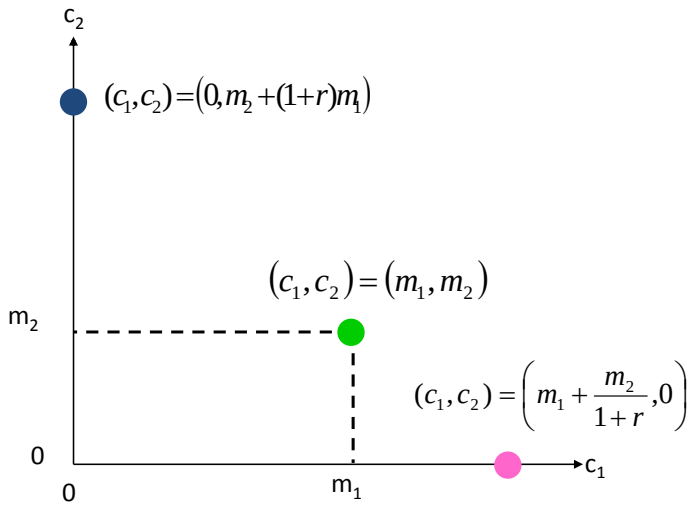
$$(c_1, c_2) = (m_1, m_2)$$

- If all period 1 income saved for period 2:

$$(c_1, c_2) = (0, m_2 + (1+r)\, m_1)$$

- If all period 2 income borrowed in period 1:

$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0 \right)$$



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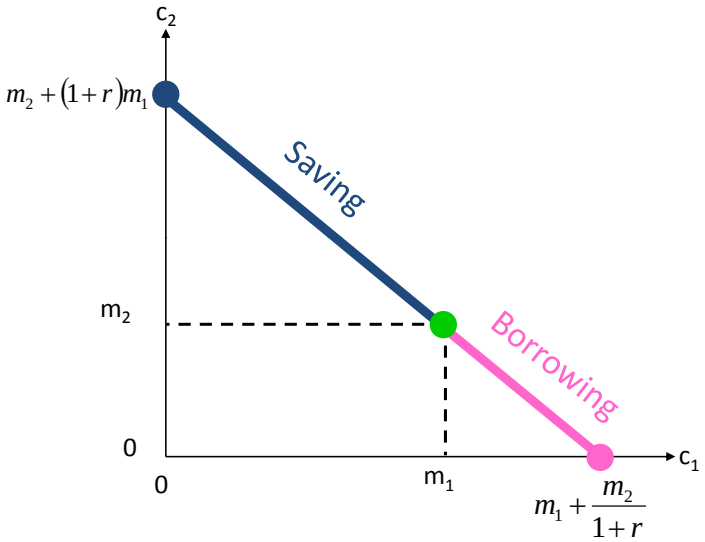
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Intertemporal Budget Constraint

- Given a period 1 consumption of c_1, period 2 consumption is

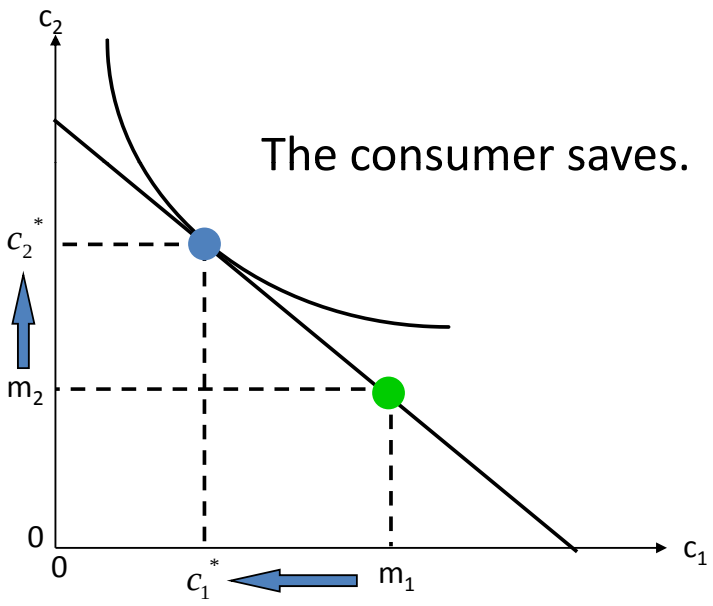
$$c_2 = \underbrace{m_2 + (1+r)\, m_1}_{\text{intercept}} \underbrace{-(1+r)}_{\text{slope}} c_1$$

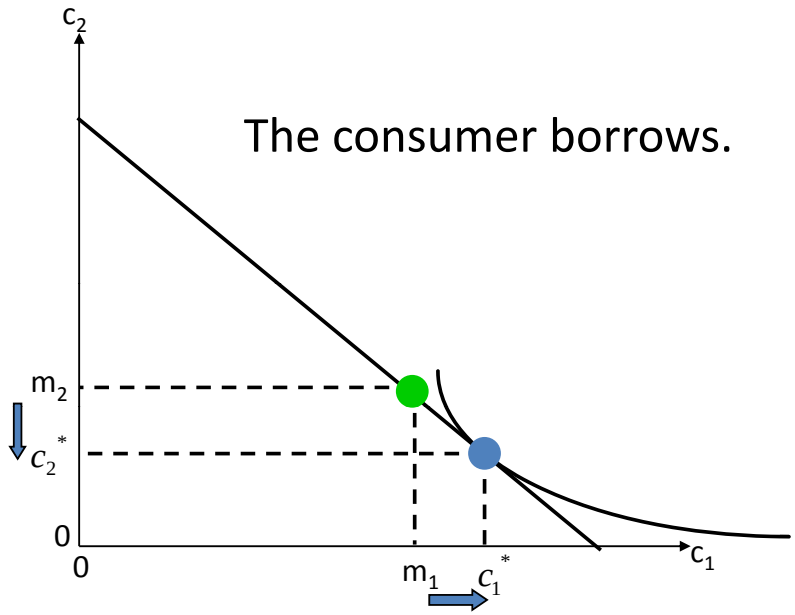


- Intertemporal budget constraint:
  - Future-valued form: $(1+r)\, c_1 + c_2 = m_2 + (1+r)\, m_1$
  - Present-valued form: $c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$

Intertemporal Choice

- Optimal intertemporal consumption bundle given by tangency point of intertemporal indifference curves and intertemporal budget constraint:





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Comparative Statics: Slutsky equation re-revisited

- The Slutsky equation for the change in c_1 due to a change in p_1 is the same as the one seen in topic 1:

$$\frac{dc_1}{dp_1} = \underbrace{\frac{\partial c_1^s}{\partial p_1}}_{(-)} + (m_1 - c_1)\underbrace{\frac{\partial c_1}{\partial m}}_{(+)}.$$

- Since a change in r is equivalent to a change in p_1, the Slutsky equation is exactly the same.
  - If $r \uparrow$, the substitution effect (the first term in the equation above) is negative; if $r \downarrow$ the substitution effect is positive.
  - The sign of the total income effect (the second term in the equation above) depends on whether the consumer is a saver or a borrower:
    - If borrower ($c_1 > m_1$), total income effect is negative.
    - If saver, ($c_1 < m_1$), total income effect is positive.
- Note: effects of $r \downarrow$ are the opposite as effects of $r \uparrow$.

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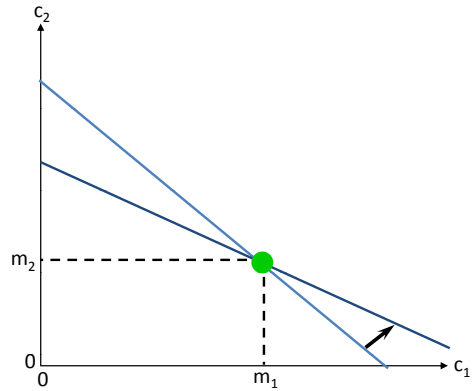
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Comparative Statics: Interest rate decrease

- Graphically, since slope budget constraint curve is $-(1+r)$,

$$r \downarrow \Rightarrow \text{flattening budget constraint.}$$



- Effects of $r \downarrow$ on optimal intertemporal consumption bundle:
  - Substitution effect: increase in cost future consumption relative to present consumption.
  - Total income effect:
    - If saver, total income effect is negative.
    - If borrower, total income effect is positive.

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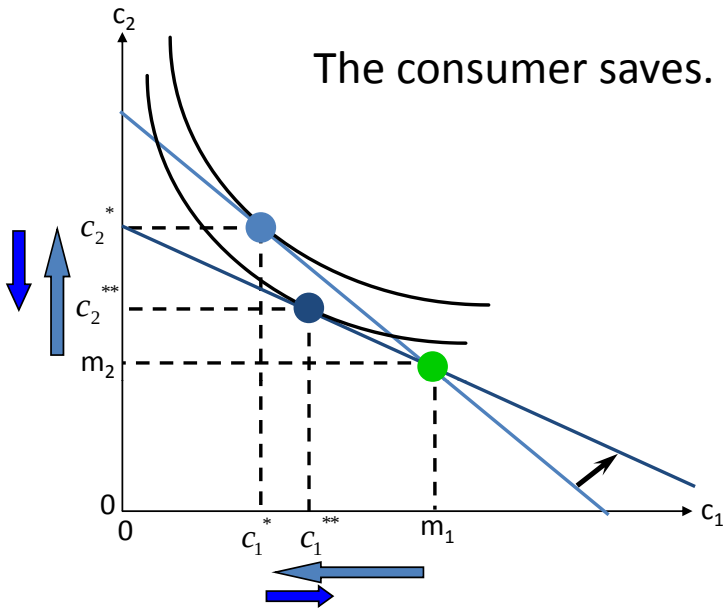
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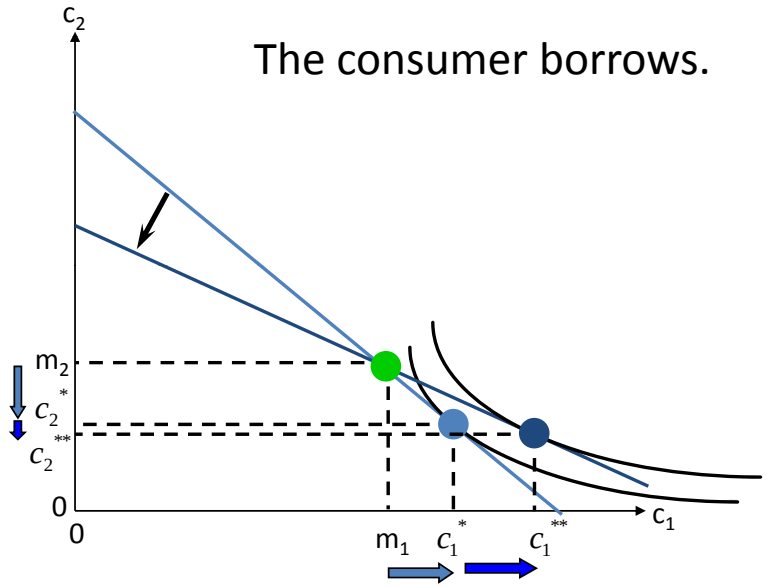
Comparative Statics: Interest rate decrease

- Total effect:

If saver, c_1?, $c_2 \downarrow$

If borrower, $c_1 \uparrow$, c_2?





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Inflation

Definitions

The ***inflation rate*** is the rate at which the level of prices for goods increases. It is equal to

$$\pi = \frac{p_2 - p_1}{p_1} \Leftrightarrow 1 + \pi = \frac{p_2}{p_1}$$

The ***real-interest rate***, ρ, is an interest rate adjusted to remove the effects of inflation. It is equal to

$$1 + \rho = \frac{1 + r}{1 + \pi} \Leftrightarrow \rho = \frac{r - \pi}{1 + \pi}$$

and, if the inflation rate is small, it can be approximated by the difference between the interest rate and the inflation rate:

$$\rho \approx r - \pi.$$

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Case II: Inflation , $p_2 = (1+\pi)\,p_1$

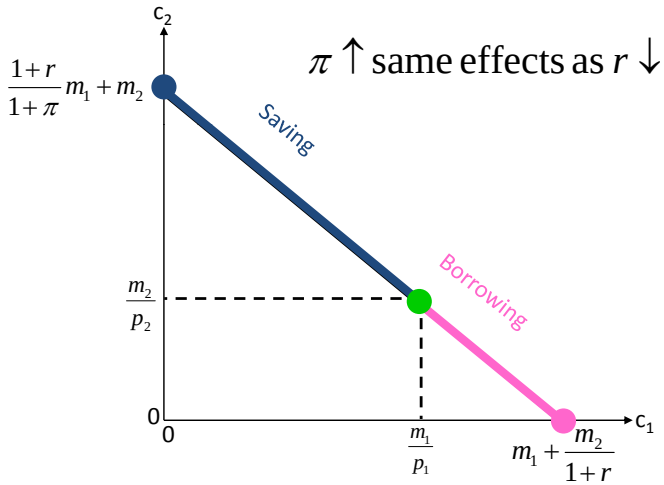
- Intertemporal budget constraint with inflation:

$$p_1 c_1 + \frac{p_2 c_2}{(1+r)} = p_1 m_1 + \frac{p_2 m_2}{(1+r)}$$

$$\Leftrightarrow c_1 + \frac{c_2}{1+\rho} = m_1 + \frac{m_2}{1+\rho}$$

- Intertemporal budget constraint curve:

$$c_2 = \underbrace{(1+\rho)\,m_1 + m_2}_{\text{intercept}} \underbrace{-(1+\rho)}_{\text{slope}} c_1$$



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Valuing Financial Securities

Definition

A ***financial security*** is a financial instrument that promises to deliver an income stream.

- Example:
  - Consider a security that pays m_1 at the end of period 1, m_2 at the end of period 2, and m_3 at the end of period 3.
  - What is the most that should be paid now for this security? The present value of this security!

$$PV = \frac{m_1}{(1+r)} + \frac{m_2}{(1+r)^2} + \frac{m_3}{(1+r)^3}.$$

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Valuing Bonds

Definition

A ***bond*** is a type of security that pays a fixed amount x for T years (its maturity date) and then pays its face value F.

Definition

Year	1	2	...	$T-1$	T
Income paid	x	x	...	x	F
Present Value	$\frac{x}{(1+r)}$	$\frac{x}{(1+r)^2}$	...	$\frac{x}{(1+r)^{T-1}}$	$\frac{F}{(1+r)^T}$

- The value of the bond is its present value:

$$PV = \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ... + \frac{x}{(1+r)^{T-1}} + \frac{F}{(1+r)^T}$$

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Outline Part I. Consumer Theory Applications

1. Topic 0. Consumer Theory Review
2. Topic 1. Buying and Selling
3. Topic 2. Intertemporal Choice
4. Topic 3. Choice under Uncertainty
   - 4.1 State-contingent budget constraints
   - 4.2 Preferences under uncertainty
   - 4.3 Insurance
   - 4.4 Risk spreading

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TOPIC 3. CHOICE UNDER UNCERTAINTY

- So far, dynamic problems considered had no uncertainty.
- However, in the real world people often make decisions with uncertainty about future prices, future wealth, or other agents' decisions.
- In this section, we study the choice problem under uncertainty using a two-state version of our consumer's choice model.
- Optimal responses: insurance purchase, risk diversification.
- Example:
  - 2 possible states of nature: car accident (loss of L€), no car accident.
  - Probabilities for each state: π_a, π_{na}.
  - Insurance: get K€ if accident by paying γK€ as insurance premium.

State-contingent budget constraints

Definitions

A contract is ***state contingent*** if it is implemented only when a particular state of Nature occurs.

A ***state-contingent consumption plan*** specifies the consumption to be implemented when each state of Nature occurs.

- Example:
  - Consumption if no accident: $c_{na} = M - \gamma K$
  - Consumption if accident: $c_a = M - L - \gamma K + K$. Hence, $K = \frac{c_a - M + L}{1-\gamma}$ and

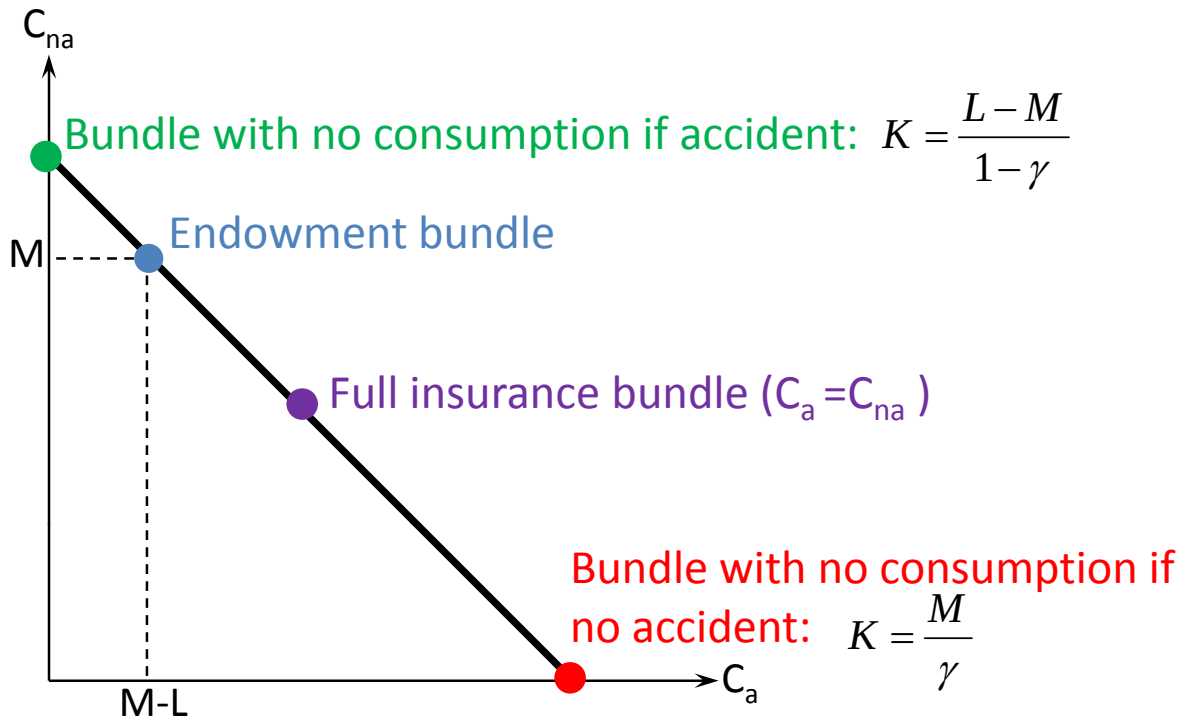
$$c_{na} = M - \gamma\left(\frac{c_a - M + L}{1-\gamma}\right) = \frac{M - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma}c_a$$

State-contingent budget constraints

Car Insurance example

- State-contingent budget constraint in car insurance example:

$$c_{na} = \underbrace{\frac{m-\gamma L}{1-\gamma}}_{\text{intercept}} \underbrace{-\frac{\gamma}{1-\gamma}}_{\text{slope}} c_a$$



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Preferences under Uncertainty

- To know what is the agents' choice, we need to know their preferences about the different state-contingent consumption plans.
- Utility across state-contingent consumption plans is a function of the consumption levels and probabilities at each state, $U(c_1, c_2, \pi_1, \pi_2)$.

Definition

A utility function $U(c_1, c_2, \pi_1, \pi_2)$ satisfies the ***expected utility*** or ***von Neumann–Morgenstern*** property if it can be written as the weighted sum of the utility at each state, where the weights are the probabilities of each state:

$$U(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

It satisfies the independence property, which means that the utility in a given state is independent of the utility in other states.

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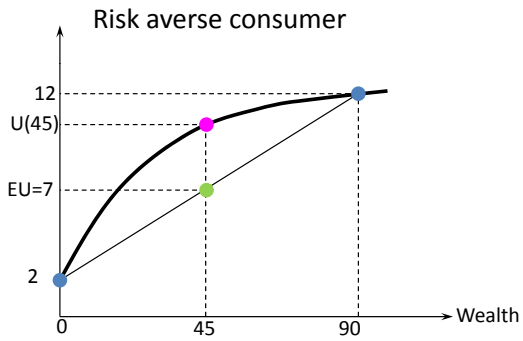
Diversification

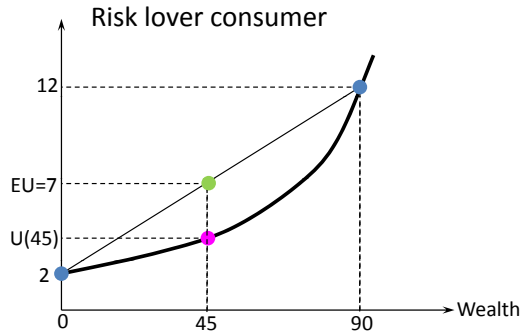
Risk aversion

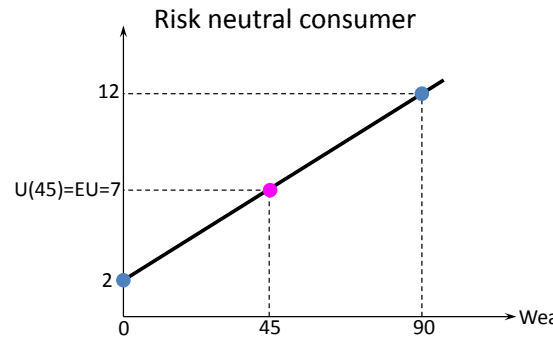
Definition

We say an agent is ***risk averse*** if the expected utility of wealth is lower than the utility of expected wealth, ***risk lover*** if it is higher, and ***risk neutral*** if it is equal.

- Example:
  - Lottery: 90€ with probability 1/2, 0€ with prob 1/2.
  - Utility levels: U(\$90) = 12, U(\$0) = 2.
  - Expected utility: EU=1/2*12+1/2*2=7.
  - Expected money value: EM=1/2*90+1/2*0=45.





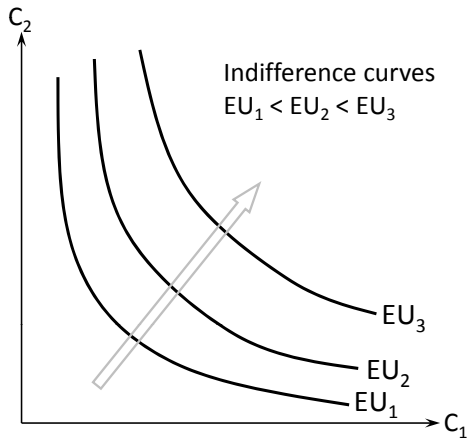


Indifference Curves

- State-contingent consumption plans that give equal expected utility are equally preferred and on the same indifference curve.
- Slope of indifference curves:

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2) \Rightarrow dEU = \pi_1 \frac{\partial U(c_1)}{\partial c_1} dc_1 + \pi_1 \frac{\partial U(c_2)}{\partial c_2} dc_2$$

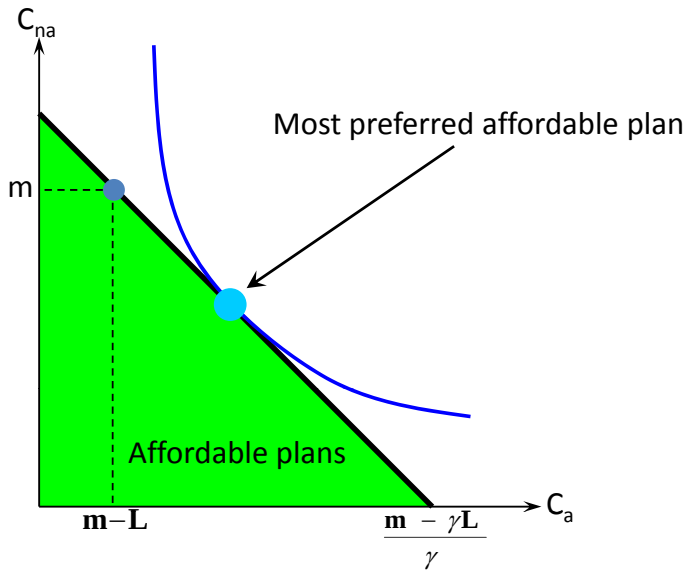
$$\pi_1 \frac{\partial U(c_1)}{\partial c_1} dc_1 + \pi_2 \frac{\partial U(c_2)}{\partial c_2} dc_2 = 0 \Rightarrow \frac{dc_2}{dc_1} = -\frac{\pi_1 \partial U(c_1)/\partial c_1}{\pi_2 \partial U(c_2)/\partial c_2}$$



Choice under uncertainty

- The optimal choice under uncertainty is the most preferred affordable state-contingent consumption plan.
- In the car insurance example, the optimal consumption plan is where the slope of indifference curves is tangent to the budget constraint:

$$\frac{\pi_a \partial U(c_a)/\partial c_a}{\pi_{na} \partial U(c_{na})/\partial c_{na}} = \frac{\gamma}{1-\gamma}$$



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Insurance

Fair Insurance

Definition

We say an insurance is ***fair or competitive*** if the expected economic profit of the insurer is zero, or, equivalently, if the price of a 1€ insurance is the probability of the insured state.

- Car insurance example:

$$\underbrace{\gamma K}_{\text{revenues}} \quad \underbrace{-\pi_a K - (1-\pi_a)0}_{\text{expected expenditures}} \quad = 0 \Rightarrow \gamma = \pi_a$$

- If the insurance is fair, the optimal choice of risk-averse consumers is full insurance:

$$\frac{\pi_a}{\pi_{na}} \frac{\partial U(c_a)/\partial c_a}{\partial U(c_{na})/\partial c_{na}} = \frac{\pi_a}{1-\pi_a} \Rightarrow \frac{\partial U(c_a)}{\partial c_a} = \frac{\partial U(c_{na})}{\partial c_{na}}$$

Hence, for risk averse consumers, $c_a = c_{na}$.

Insurance

Unfair Insurance

Definition

We say an insurance is ***unfair*** if the insurer makes positive expected economic profits.

- If the insurance is unfair, the optimal choice of risk-averse consumers is less than full insurance:

$$\underbrace{\gamma K}_{\text{revenues}} \quad \underbrace{-\pi_a K - (1-\pi_a)0}_{\text{expected expenditures}} > 0 \Rightarrow \frac{\gamma}{1-\gamma} > \frac{\pi_a}{\pi_{na}}$$

Hence, $\frac{\pi_a}{\pi_{na}} \frac{\partial U(c_a)/\partial c_a}{\partial U(c_{na})/\partial c_{na}} = \frac{\gamma}{1-\gamma}$ implies that

$$\frac{\partial U(c_a)}{\partial c_a} > \frac{\partial U(c_{na})}{\partial c_{na}}$$

so, for risk averse consumers, $c_a < c_{na}$.

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Diversification

- Asset diversification typically lowers (or keeps) expected earnings in exchange for lowered risk. This is going to be the case as long as the asset prices are not perfectly correlated across states.
- Example: two firms, two states (prob. 1/2), agent with 100€ to spend in firms' share.
  - Firm A: shares' cost 10€, profits per share in state 1 100€, in state 2 20€.
  - Firm B: shares' cost 10€, profits per share in state 1 20€, in state 2 100€.

	10 shares of A	10 shares of B	5 of A, 5 of B
Profits in 1	1000€	200€	600€
Profits in 2	200€	1000€	600€
Expected profits	600€	600€	600€