Part II. General Equilibrium and Welfare Theorems

Endowments

Exchange

Robinson Crusoe

Production

Microeconomics IV

Part II. General Equilibrium and Welfare Theorems

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PART II. General Equilibrium and Welfare Theorems

- So far, partial equilibrium analysis:
  - Each market studied independently;
  - only own price changes taken into account;
  - ignoring effects of changes in other markets.
- In general, however, demand and supply of different markets interact with each other:
  - Goods may be complements or substitutes;
  - Consumers' income may depend on prices of many markets.
- In this part, we study the equilibrium of all markets simultaneously.
  - Simplifications: perfect competition, one or two goods.
  - First, pure exchange economies.
  - Second, introduction of production decisions.

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2. General equilibrium in a pure exchange economy
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TOPIC 5. General equilibrium in an exchange economy

- So far, consumers' income taken as exogenous and independent of prices. In reality, consumers' income coming from exchange by sellers and buyers.
- How are incomes generated? How does the value of income depend upon commodity prices?
- How can we put all this together to explain better how price changes affect demands?

Part II. General Equilibrium and Welfare Theorems

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5.1 Consumers with initial endowments

- In this chapter, consumers get income from endowments. This makes the budget set definition change slightly.

Definition

The list of resource units with which a consumer starts is her ***endowment***, denoted by $\omega = (\omega_1, \omega_2)$.

Definitions

Given p_1 and p_2, the ***budget constraint*** for a consumer with an endowment $\omega = (\omega_1, \omega_2)$ is

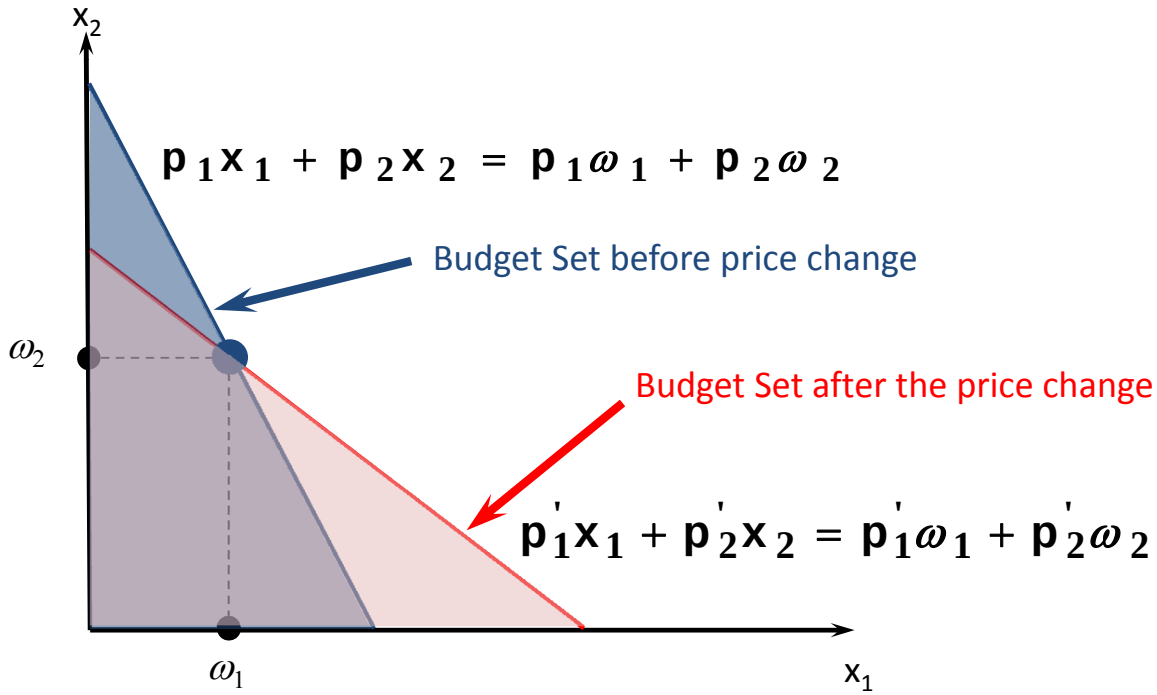
$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2.$$

and the ***budget set*** is formally defined as

$$\{(x_1, x_2) \mid x_1 \geq 0, x_2 \geq 0 \text{ and } p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2\}.$$

Endowments and Budget Sets

- Graphically, the endowment point is always on the budget constraint.
- Hence, price changes pivot the constraint around the endowment point.



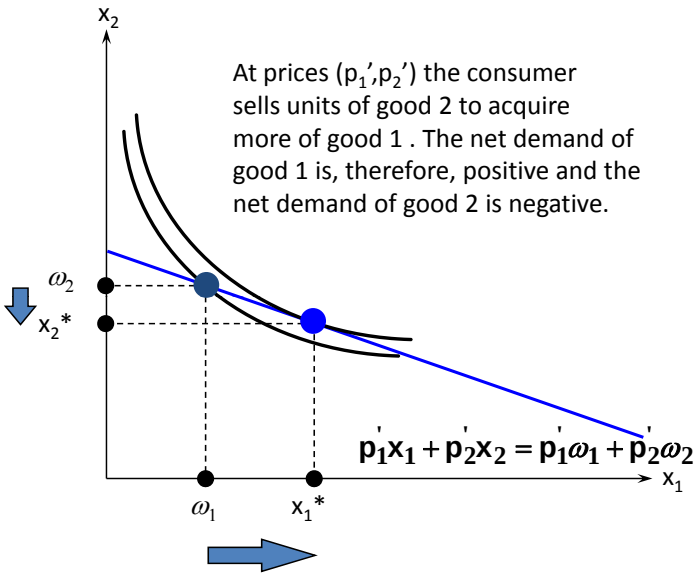
Net demand

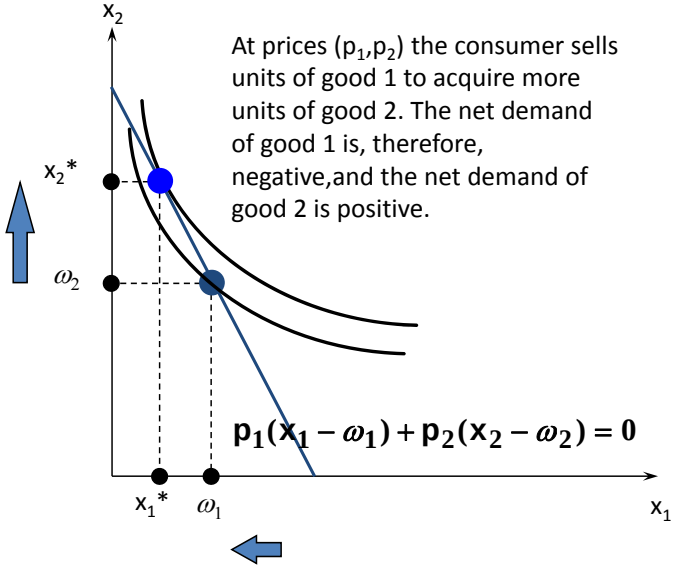
Definition

The difference between final consumption and initial endowment of a given good i, $x_i^* - \omega_i$, is called ***net demand of good*** i.

The sum of the values of net demands is zero:

$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$$
$$\Leftrightarrow p_1 (x_1 - \omega_1) + p_2 (x_2 - \omega_2) = 0.$$





Price offer curve

Definition

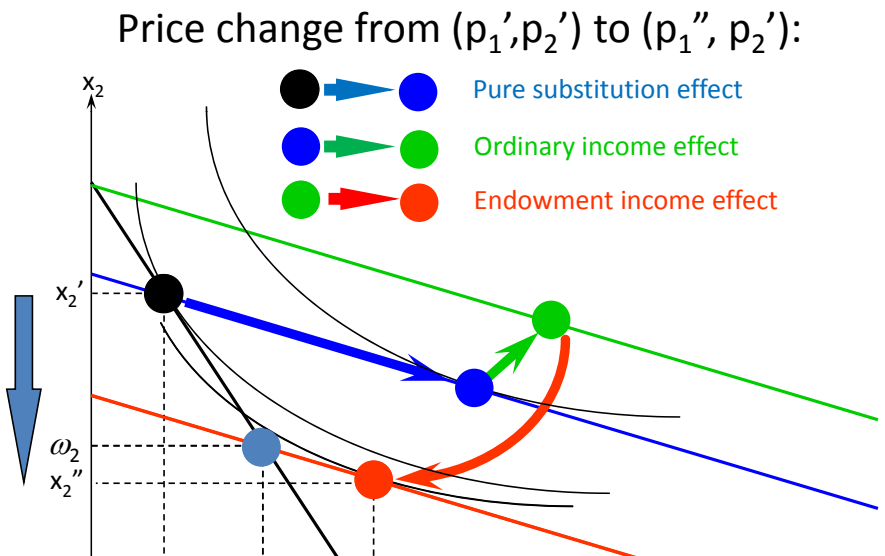
Price-offer curve contains all the utility-maximizing gross demands for which the endowment is exchanged.



Slutsky equation revisited

In an endowment economy, the overall change in demand caused by a price change is the sum of *a pure substitution effect*, *an (ordinary) income effect*, ***and an endowment income effect***.

- *Pure Substitution Effect*: effect of relative prices change.
- *Income Effect*: effect of original bundle cost change.
- ***Endowment Income Effect:*** change in demand due only to the change in endowment value.



Slutsky equation revisited (2)

Overall change in demand of normal good (demand increases with income) caused by own price change:

- When income is exogenous, both the substitution and (ordinary) income effects increase demand after an own-price fall; hence, a normal good's ordinary demand curve slopes down (thus, Law of Downward-Sloping Demand always applies to normal goods when income is exogenous).

- When income is given by initial endowments, endowment-income effect decreases demand if consumer supplies that good (negative net demand); thus, if the endowment income effect offsets the substitution and the (ordinary) income effects, the demand function could be upward-sloping!

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5.2 General equilibrium in a pure exchange economy

- Two consumers, A and B, two goods, 1 and 2.
- Endowments of goods 1 and 2:

$$\omega^A = \left(\omega_1^A, \omega_2^A\right), \ \omega^B = \left(\omega_1^B, \omega_2^B\right).$$

- Given a price vector (p_1, p_2), consumers choose their favorite affordable allocation (as in topic 1):

$$p_1 x_1^A + p_2 x_2^A \leq p_1 \omega_1^A + p_2 \omega_2^A$$
$$p_1 x_1^B + p_2 x_2^B \leq p_1 \omega_1^B + p_2 \omega_2^B$$

- Prices must be such that allocations chosen are feasible:

$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B, \ x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

Part II. General Equilibrium and Welfare Theorems

Endowments

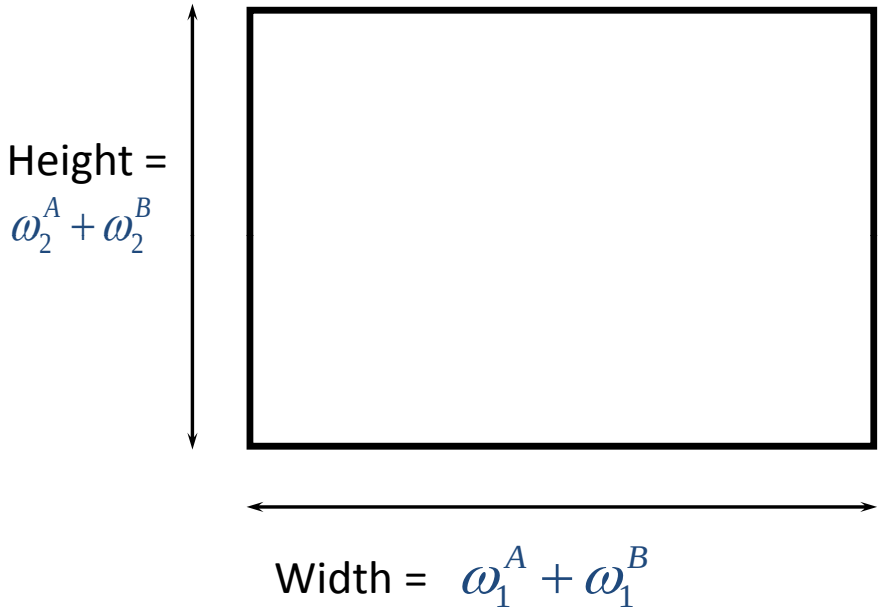
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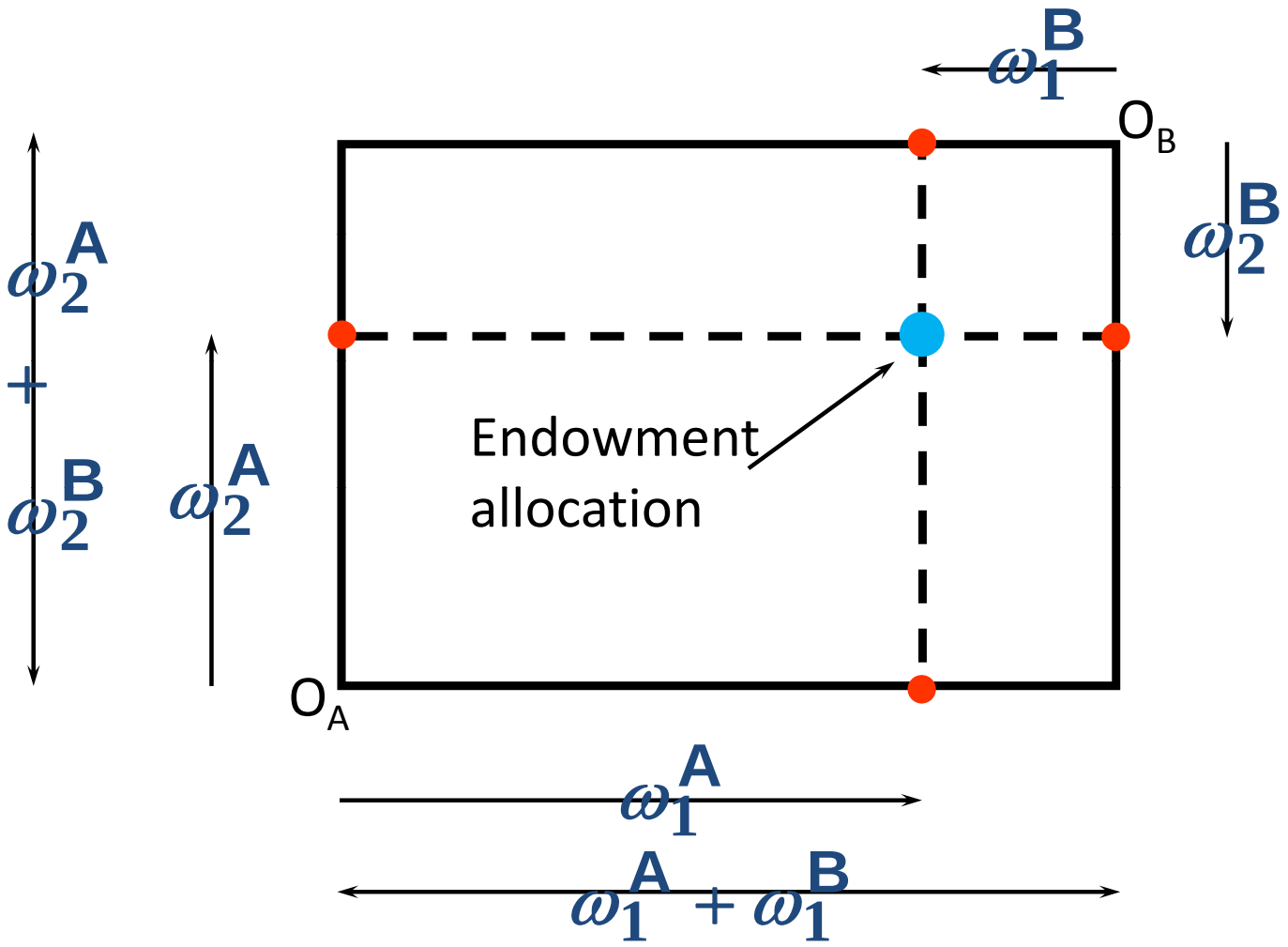
Edgeworth Box

- ***Edgeworth box*** is diagram showing all possible allocations of the available quantities of goods 1 and 2 between the two consumers.
  - The dimensions of the box are the quantities available of the goods.
  - The allocations depicted are the ***feasible allocations***.



Endowment allocation

- The ***endowment allocation*** is the before-trade allocation:

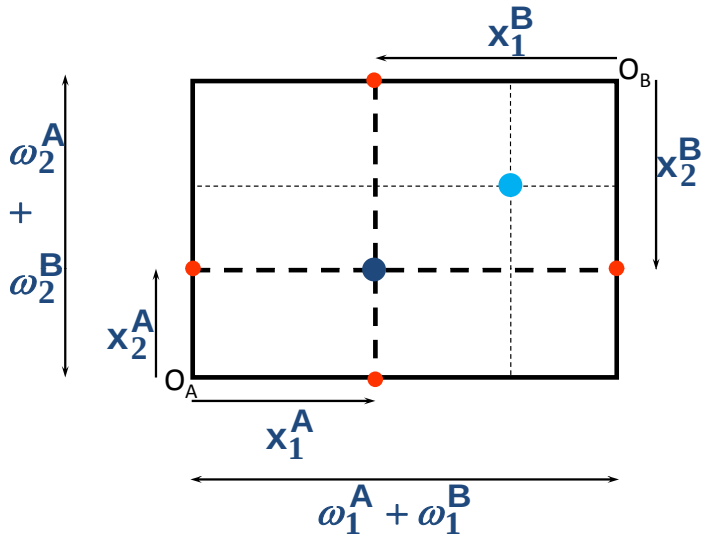


Feasible reallocations

- Which reallocation will consumers choose?
  - Feasible.
  - Pareto-improving over the endowment allocation.
- An allocation is ***feasible*** if and only if

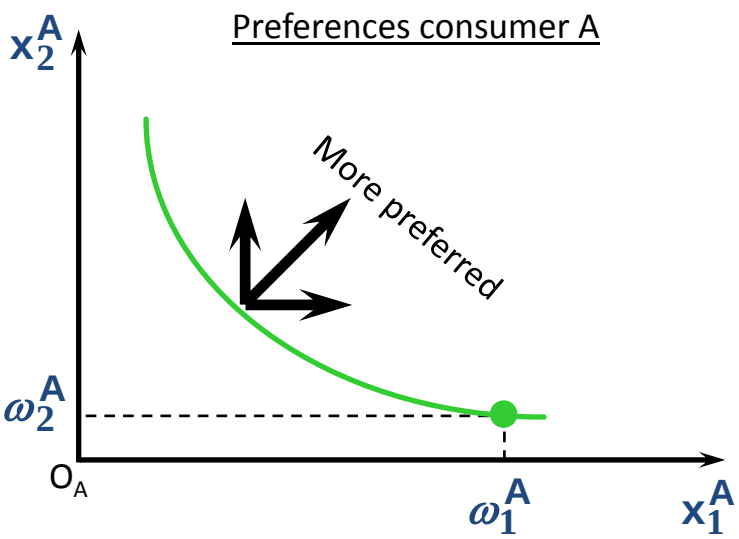
$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

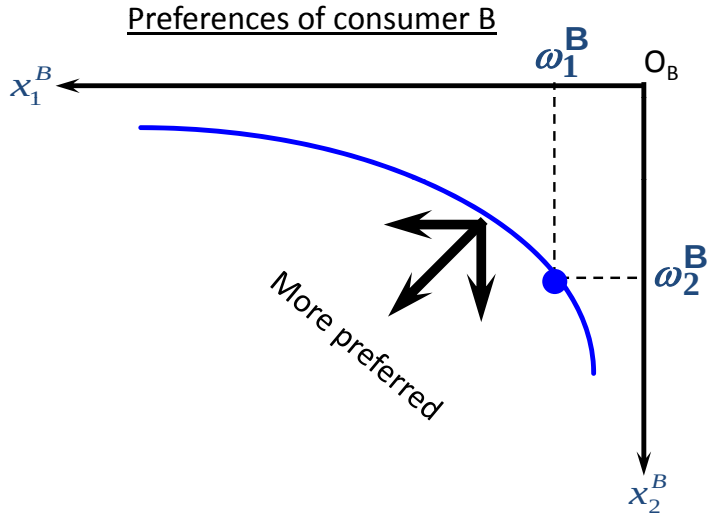
- All points in the box, including the boundary, represent feasible allocations of the combined endowments:



Pareto-improving allocations

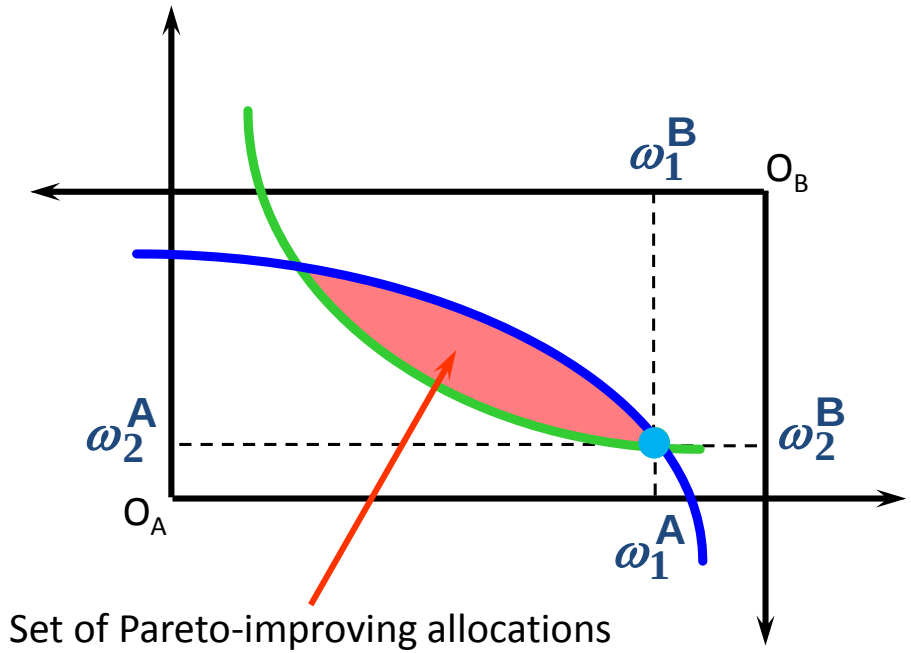
- An allocation is ***Pareto-improving*** over the endowment allocation if it improves the welfare of a consumer without reducing the welfare of another.
- Preferences of consumers A and B:





Pareto-improving allocations

- An allocation is ***Pareto-improving*** over the endowment allocation if it improves the welfare of a consumer without reducing the welfare of another.

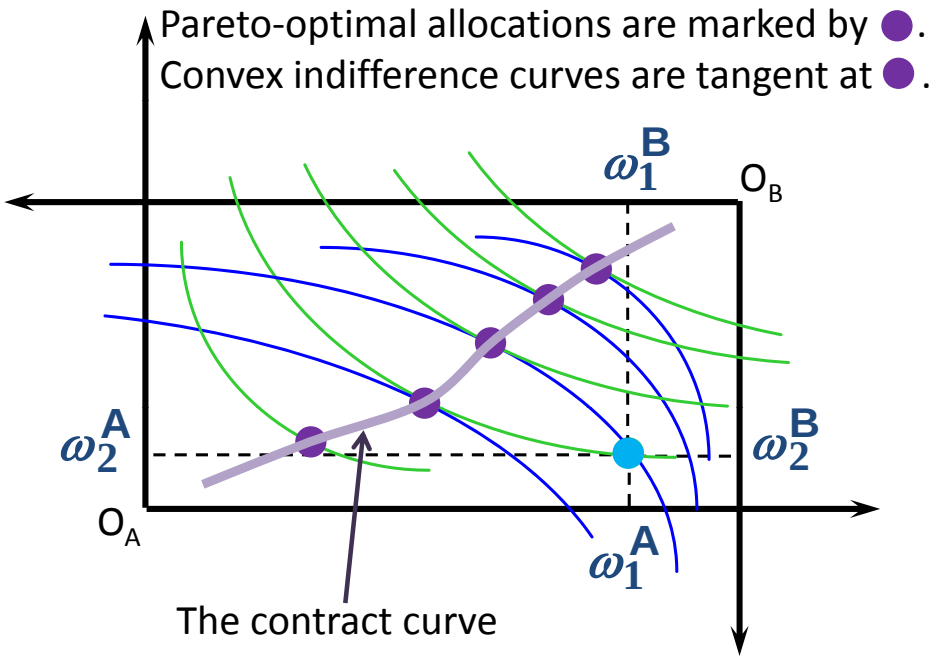


- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.

Contract curve

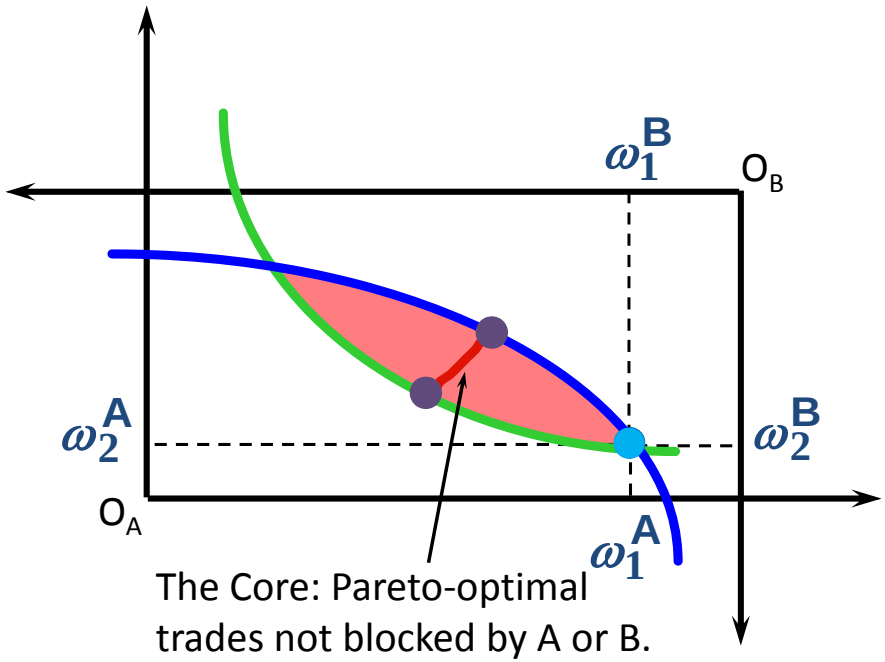
- An allocation is ***Pareto-optimal*** if the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

- The set of all Pareto-optimal allocations is called ***contract curve***.



The Core

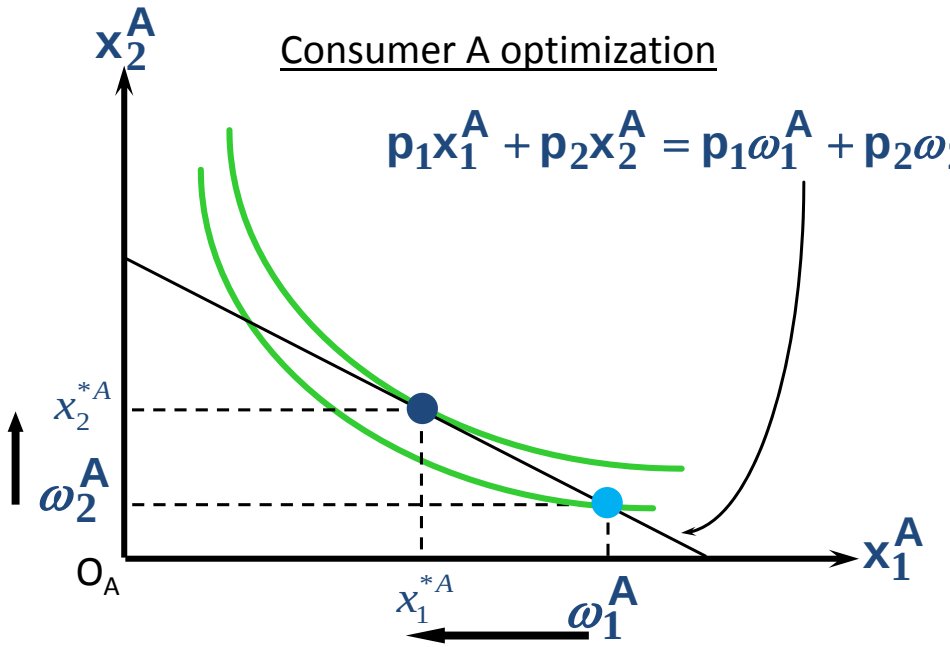
- ***The core*** is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.



- Rational trade should achieve a *core* allocation.

Trade in competitive markets

- Specific core alloation achieved depends upon the manner in which trade is conducted.
- In perfectly competitive markets, each consumer is a price-taker trying to maximize her own utility given (p_1, p_2) and her own endowment:



- Similarly for consumer B.

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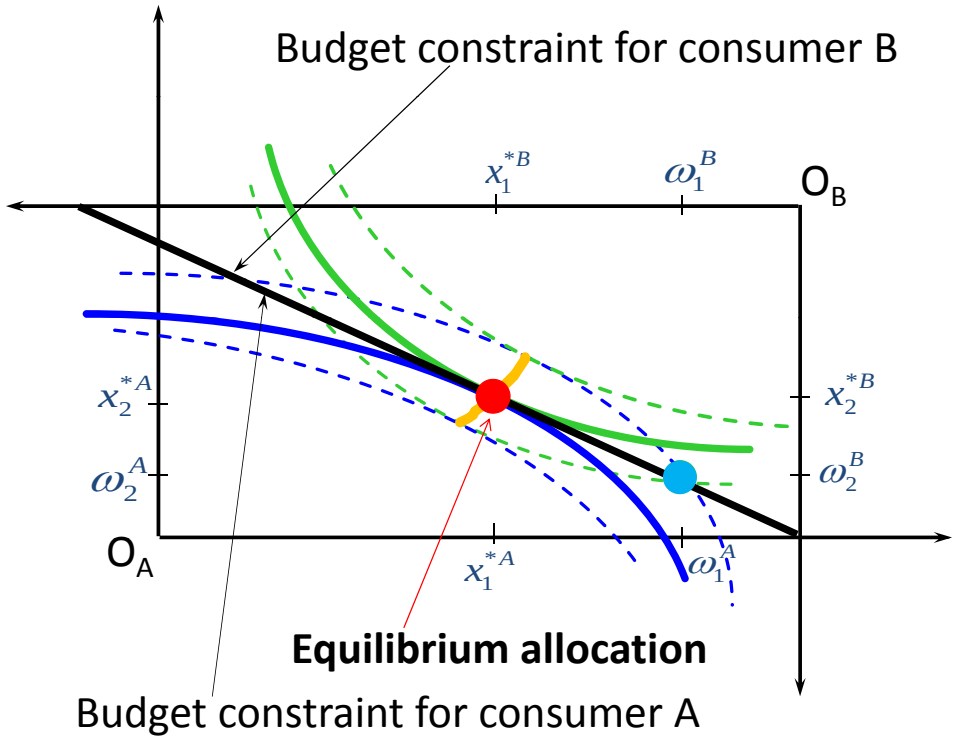
Robinson Crusoe

Production

Trade in competitive markets

- At equilibrium prices p_1 and p_2, both consumers maximize their own utility and both markets clear:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$



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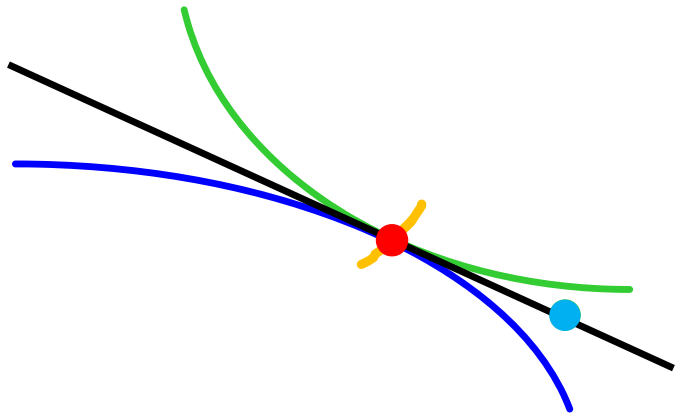
Production

First fundamental theorem of welfare economics

Theorem

Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.

- Note: Indifference curves are tangent, which implies that the equilibrium allocation is Pareto optimal.

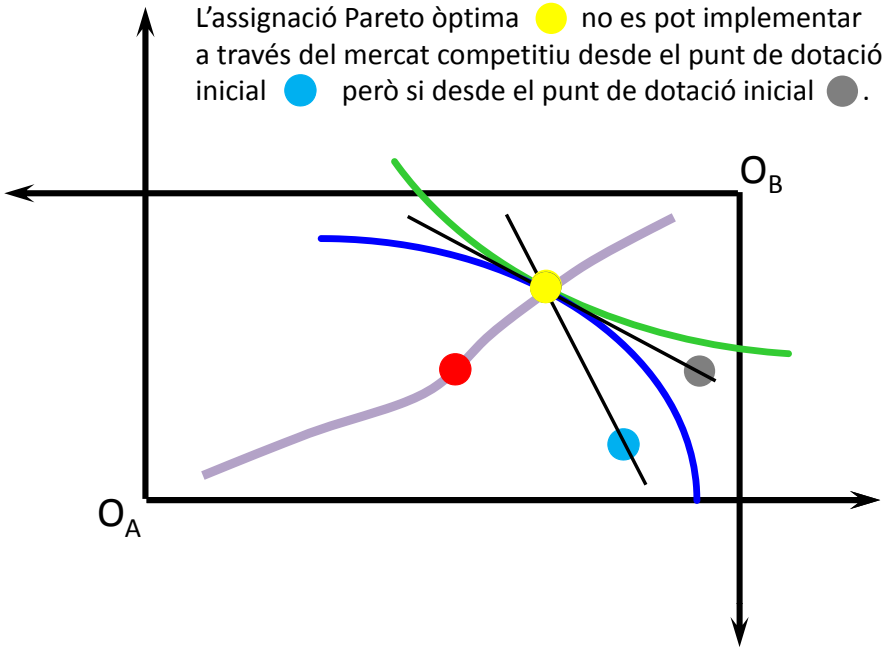


Second fundamental theorem of welfare economics

Theorem

Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation, there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.

- In other words, any Pareto-optimal allocation can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged.



Part II. General Equilibrium and Welfare Theorems

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Walras' Law

Theorem

If consumer's preferences are "well-behaved", so that for any positive prices (p_1, p_2) consumers spend all their budget, the summed market value of excess demands is zero. This is ***Walras' Law****.*

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$
$$p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B$$

$\Rightarrow$

$$p_1 \left(x_1^A + x_1^B - \omega_1^A - \omega_1^B \right) + p_2 \left(x_2^A + x_2^B - \omega_2^A - \omega_2^B \right) = 0$$

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Implications of Walras' Law

- One implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

$$\Rightarrow \text{If } x_1^A + x_1^B = \omega_1^A + \omega_1^B, \text{ then } x_2^A + x_2^B = \omega_2^A + \omega_2^B.$$

- Another implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

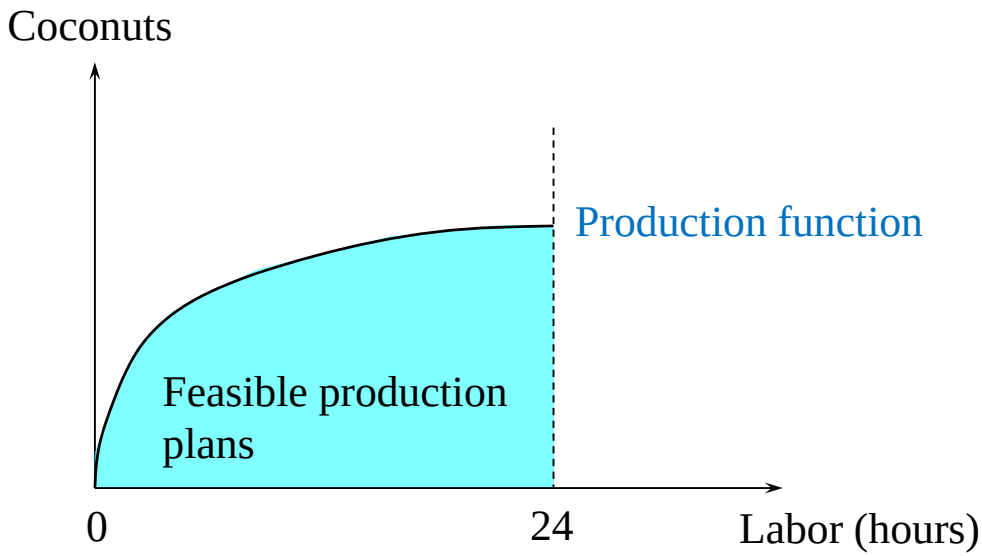
$$\Rightarrow \text{If } x_1^A + x_1^B < \omega_1^A + \omega_1^B, \text{ then } x_2^A + x_2^B > \omega_2^A + \omega_2^B.$$

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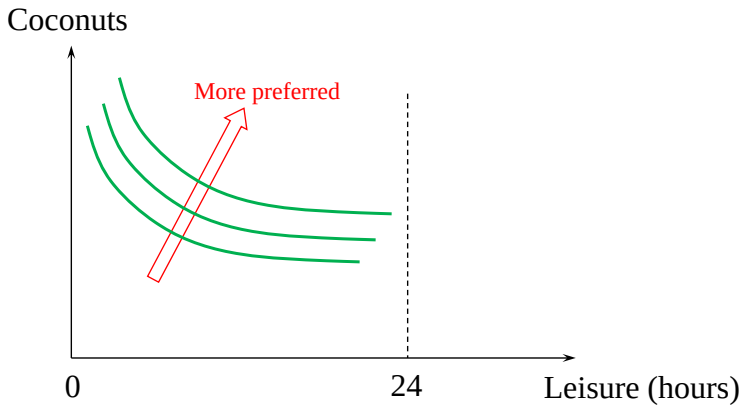
6.1 General equilibrium in a Robinson Cruose economy

- Add input and output markets, firms' technologies.
- Robinson Crusoe's Economy:
  - One agent: Robinson Crusoe.
  - Endowment: a fixed quantity of time.
  - Decision: use time for labor (production of coconuts) or leisure.
- Technology: coconuts are obtained from labor according to the production function $C = f(L)$.

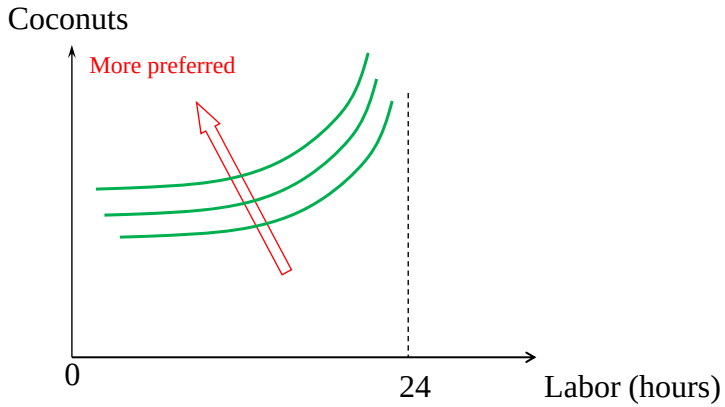


Robinson Crusoe's preferences

- Indifference curves in the leisure-coconut diagram: coconut is a good, leisure is a good:

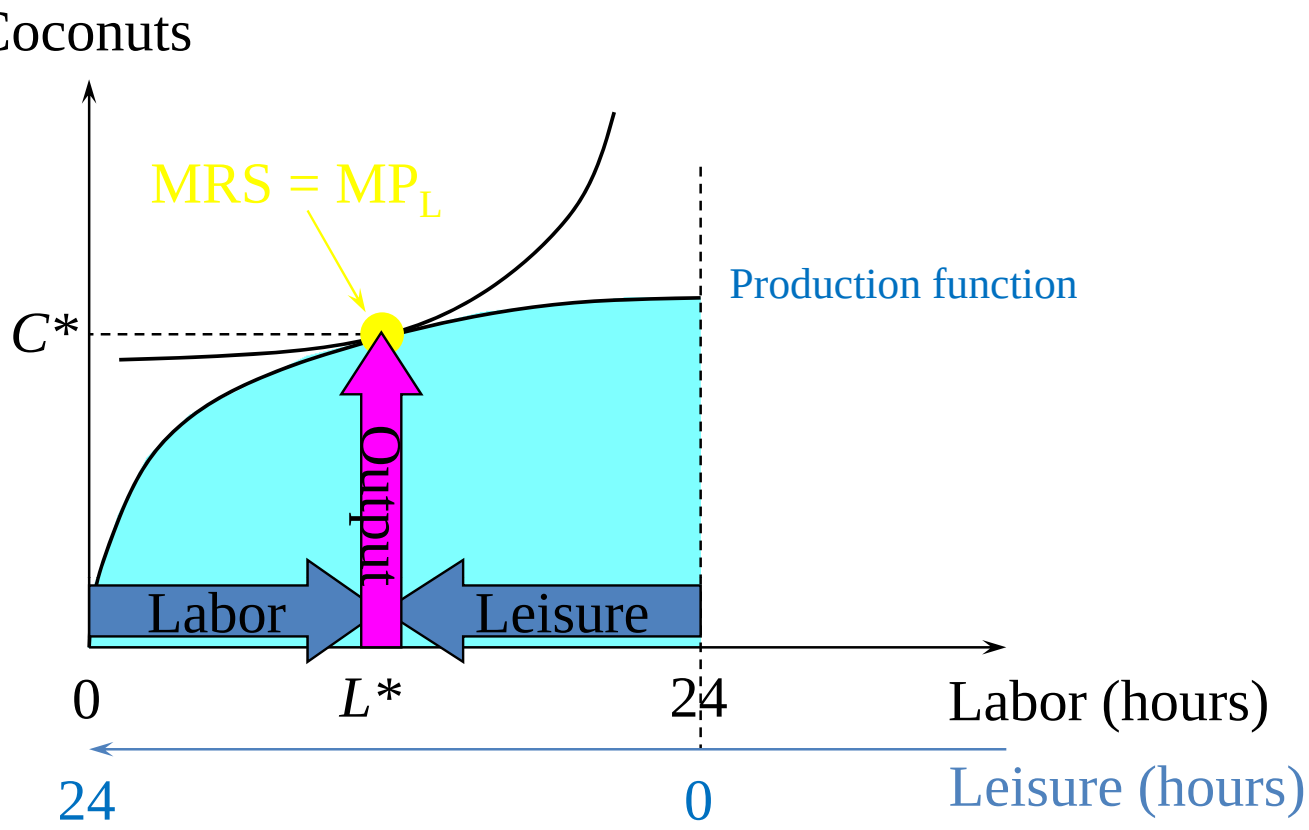


- Indifference curves in the labor-coconuts diagram: coconut is a good, labor is a bad.



Robinson Crusoe's choice

- Robinson chooses time allocation and, as a result, his consumption of coconuts:



Part II. General Equilibrium and Welfare Theorems

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Competitive equilibrium in the Robinson economy

- Robinson esquizofrenia:
  - We first consider Robinson as a ***profit-maximizing firm***, who takes prices as given and decides how much hours to hire and how much to produce.
  - Then, we consider Robinson as a ***utility-maximizing consumer*** who gets the firm profits and decides his hours of work and his consumption of coconuts.
- Let p be the coconuts price and w the wage rate.
  - Use coconuts as the numeraire good; i.e. price of a coconut $= 1$.

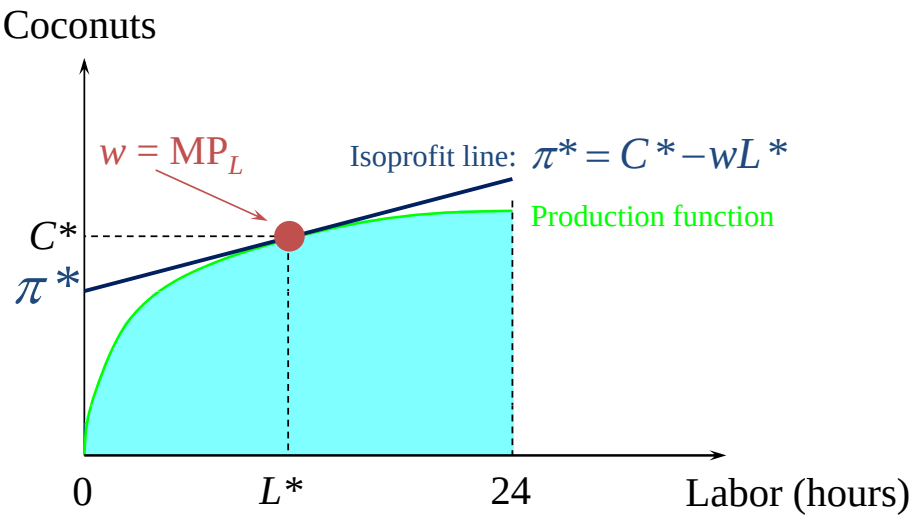
Robinson as a firm

- Optimization problem of the firm: given w, choose labor demand and coconut supply to maximize profits:

$$\max_L \pi = C - wL = f(L) - wL \Rightarrow MP(L^*) = w$$

  - Labor demanded: L^*, output supplied: $C^* = f(L^*)$.

- Graphically, firm demands L such that production function tangent to isoprofit line:



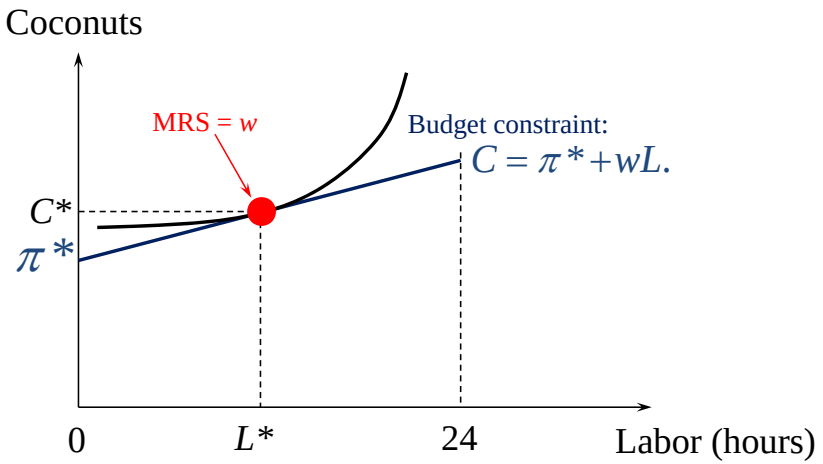
Robinson as consumer

- Optimization problem of the consumer: choose labor supply and coconut demand to maximize utility subject to the budget constraint:

$$\max_{C,L} U(C,L) \text{ s.t. } C = \pi^* + wL \Rightarrow \frac{\partial U(C,L)/\partial L}{\partial U(C,L)/\partial C} = w$$

  - Labor supplies: L^*, coconuts demanded: C^*.

- Graphically, consumer chooses C and L such that the indifference curve is tangent to the budget constraint:

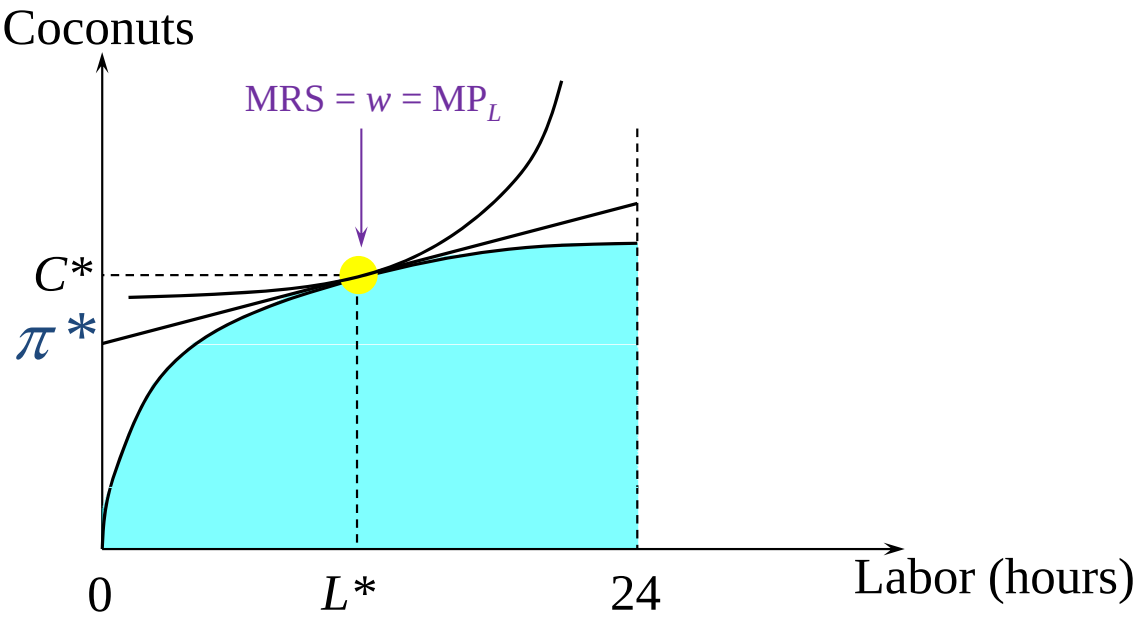


Market equilibrium

- In equilibrium, wage rate must be such that

quantity labor demanded = quantity labor supplied

(quantity output supplied = quantity output demanded)

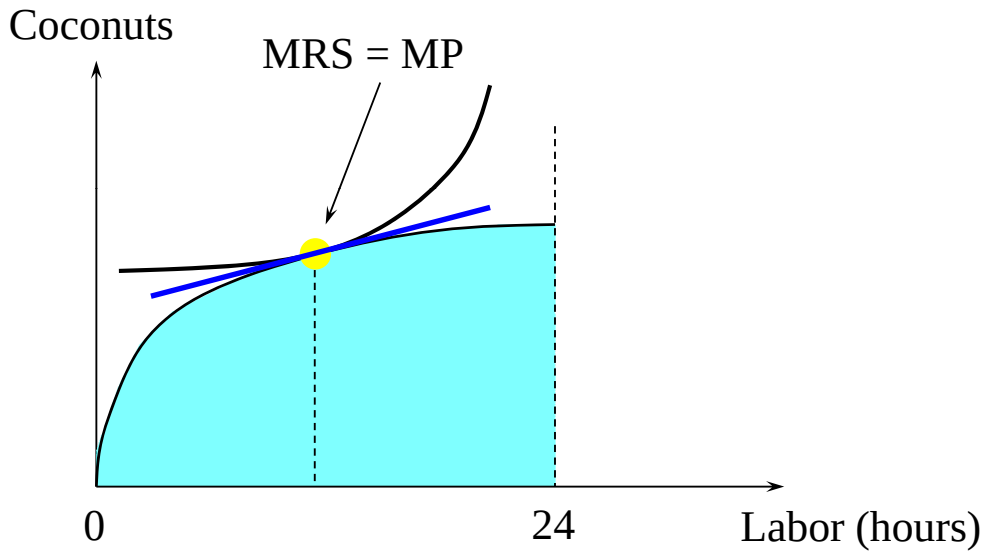


First Fundamental Theorem of Welfare Economics

Theorem

If consumers' preferences are convex and there are no externalities in consumption or production, a competitive market equilibrium is Pareto efficient.

- Pareto efficiency: $MRS = MP$:
  - Competitive equilibrium achieves Pareto efficiency: w is the common slope of the isprofit line and the budget constraint.



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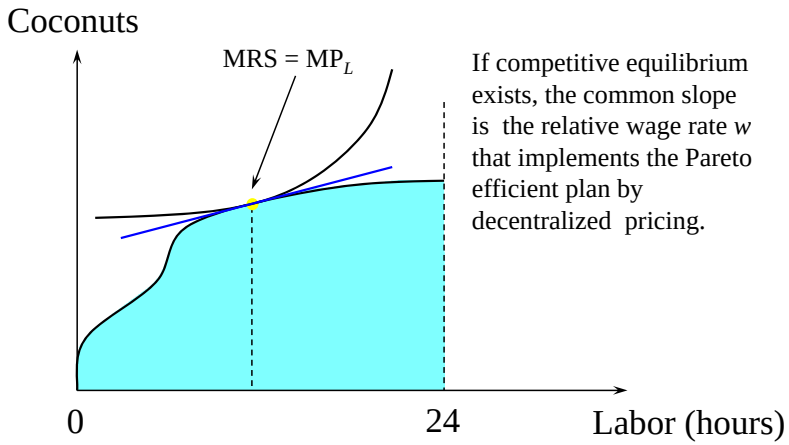
Second Fundamental Theorem of Welfare Economics

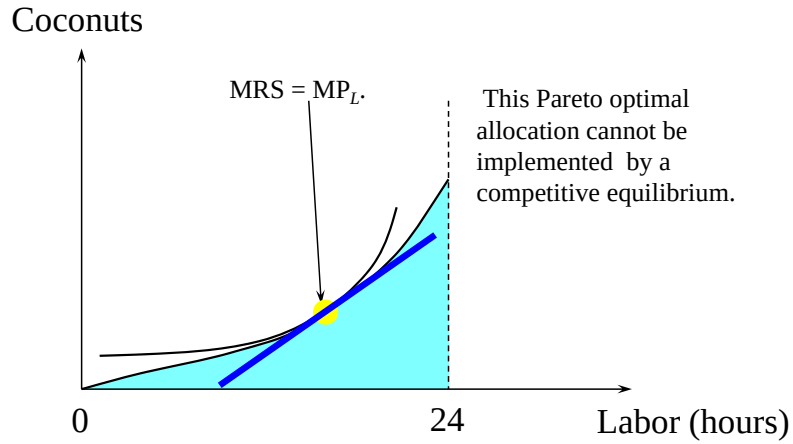
Theorem

If consumers' preferences are convex, firms' technologies are convex, and there are no externalities in consumption or production any Pareto efficient economic state can be achieved as a competitive market equilibrium.

Non-convex technologies

- The First Welfare Theorems still holds if firms have non-convex technologies since it does not rely upon firms' technologies being convex.
- The Second Welfare Theorem does not hold if firms have non-convex technologies.





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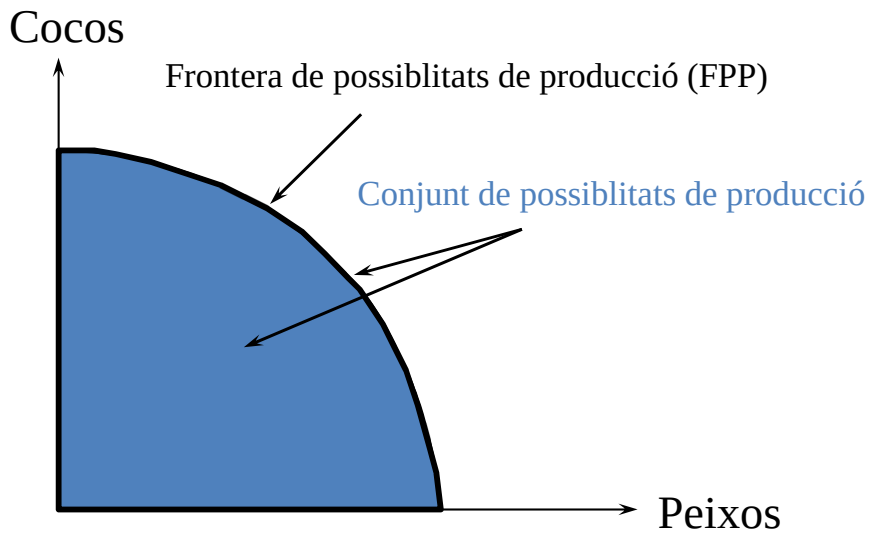
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6.2 General equilibrium in an economy with production

- In this section, introduction more inputs/goods into the GE analysis.

Definition.

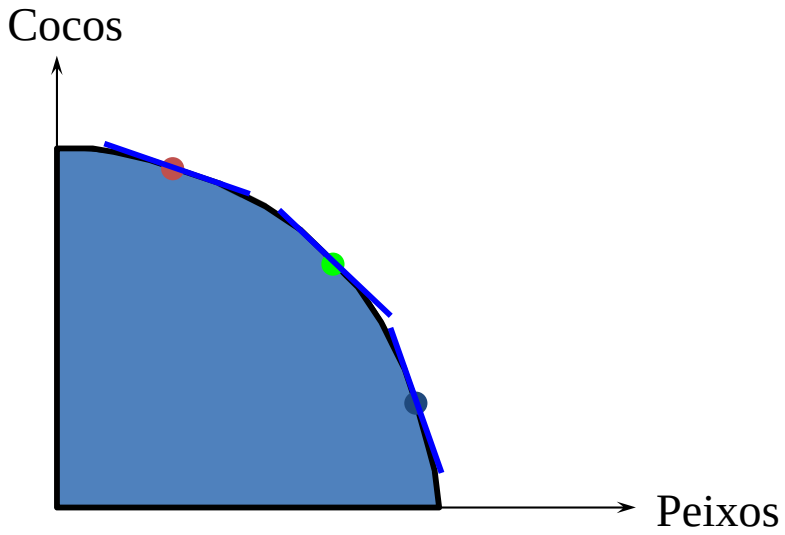
*The **Production Possibilities** set represents the production combination that are feasible given the technology. The upper bound is called **Production Possibilities Frontier (PPF)**.*



The marginal rate of transformation

Definition.

The slope of the Production Possibilities set is the ***Marginal Rate of Transformation****. Thus, it gives the extra amount we can produce of a good if we give up a unit of the other good.*



- An example:

$$C = 2L_c,\ P = 3L_p,\ L_c + L_p = 10$$

$$\Rightarrow$$

$$\frac{P}{3} + \frac{C}{2} = 10,\ RMT = -\frac{2}{3}$$

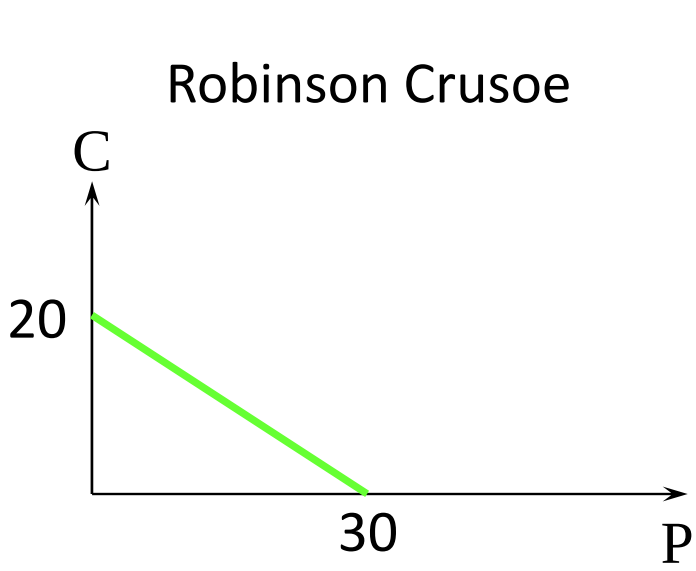
The combined production possibilities set

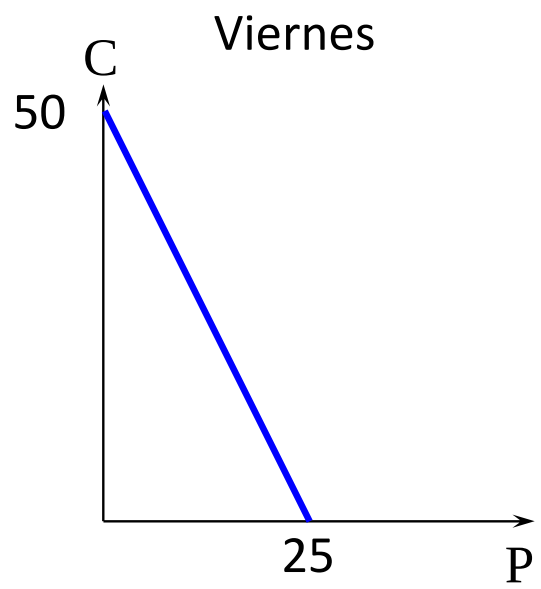
- Let's introduce a new worker in the economy: **Friday** (V).
- Suppose the abilities of Friday are the following:

$$C = 5L_c,\ P = \frac{5}{2}L_p,\ L_c + L_p = 10$$

$$\Rightarrow$$

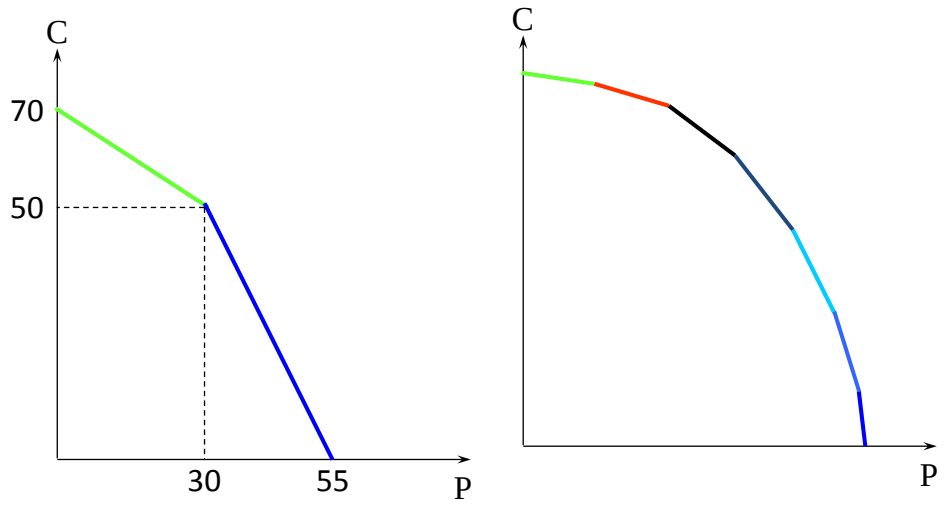
$$\frac{2P}{5} + \frac{C}{5} = 10,\ RMT = -2$$





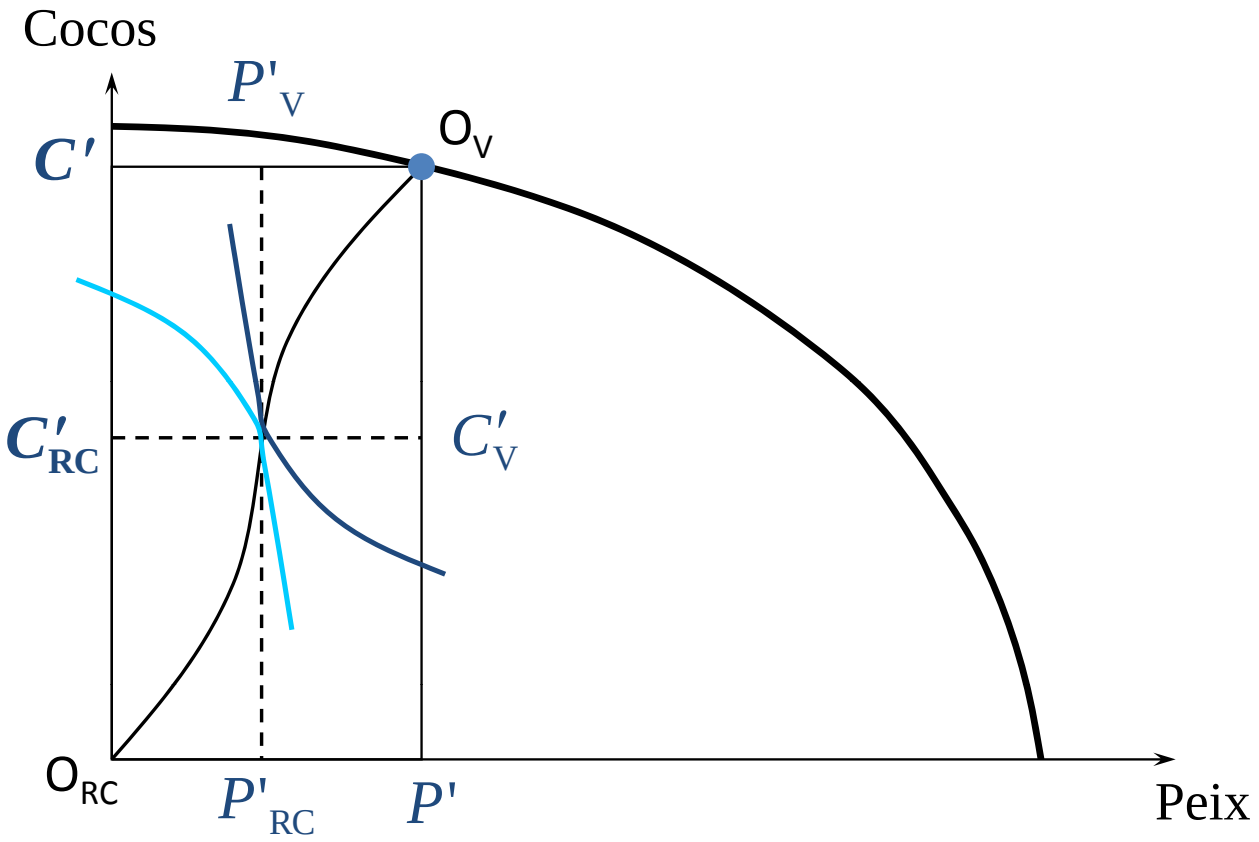
The comparative advantage

- Robinson has a comparative advantage in the production of fish, Friday on the coconuts production:
  - Robinson has to give up 2/3 of coconuts for each extra fish produced.
  - Friday has to give up 2 coconuts for each extra fish produced.
- The **combined production possibilities set** is the amount is the total feasible production by both workers given the existing technology
  - As we add more workers, the PPF keeps smoothing out:



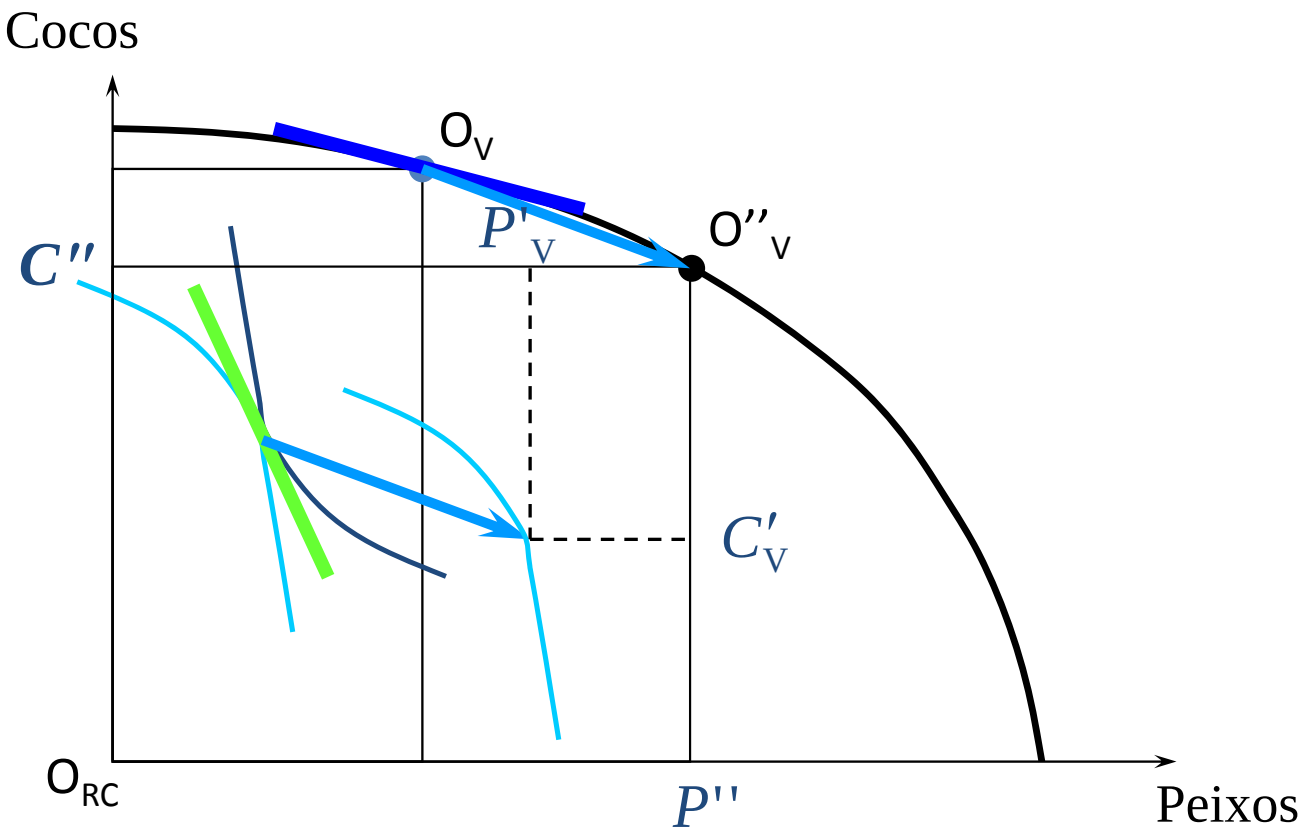
Pareto efficiency

- The PPF shows the technically efficiency allocations.
- Whare the Pareto efficient allocations?
- Given a point (P', C') on the PPF, the Pareto-efficient allocation (in terms of consumption) is the one where the indifference curves are tangent:



Pareto efficiency (2)

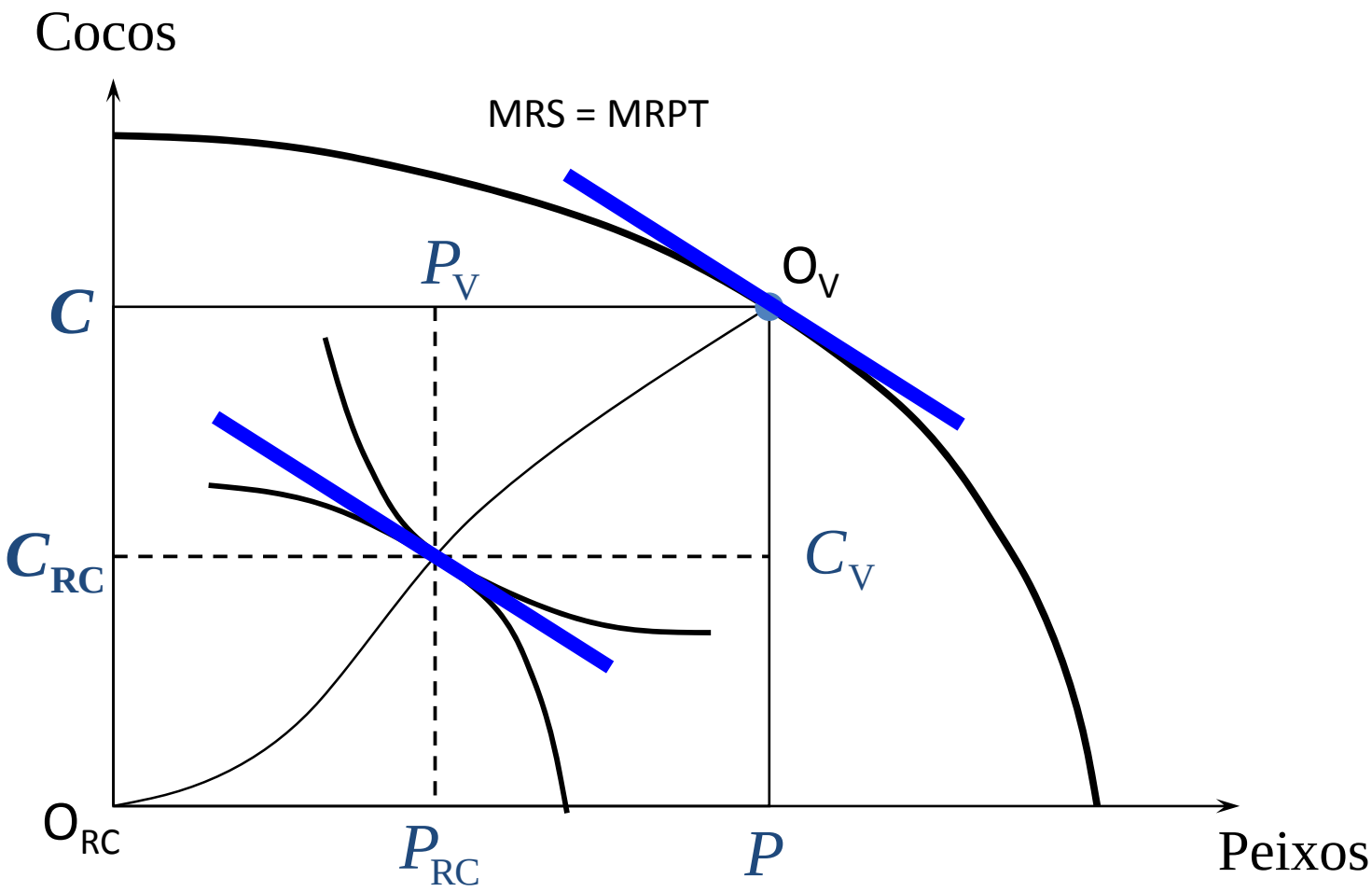
- What are the allocations that are Pareto efficient also in terms of production?
- If the MRS are not equal to the MRT, then it is possible to increase the welfare of one consumer without changing the other consumer welfare by changin the production point.



Pareto efficiency (3)

- Thus, the Pareto efficient allocations are where the MRS of the two consumers are equal to the MRT:

$$MRS_{RC} = MRS_V = MRT$$



Part II. General Equilibrium and Welfare Theorems

Endowments

Exchange

Robinson Crusoe

Production
PPF
Efficiency
Equilibrium

Competitive equilibrium

- The competitive equilibrium in this economy consists on a situation where:
  - Consumers, Robinson and Friday, maximize their utility.
  - Firms maximize their benefits.
  - All markets clear (labor, fish, coconuts).

- Optimization problems:
  - Consumers:

$$\max_{C_i, L_i} U(C_i, P_i) \text{ s.a. } p_c C_i + p_f F_i = \pi_i^* + w_i L_i,\ i = RC, F$$

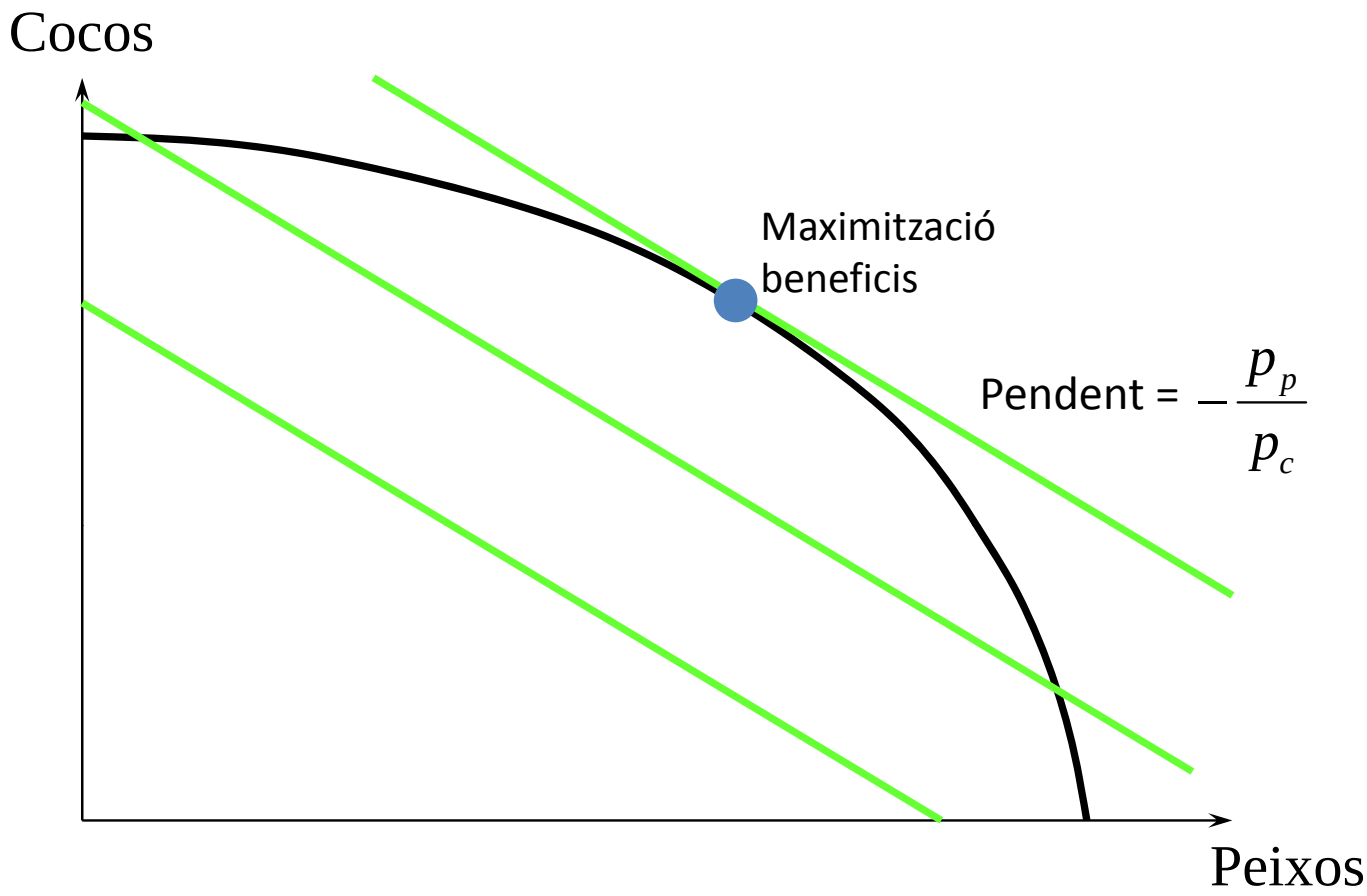
  - Producers:

$$\max_{L_{RC}, L_V} \pi = p_c C + p_f F - w_{RC} L_{RC} - w_F L_F$$

Producer optimization

- The profit maximization occurs when

$$MRT = -\frac{p_f}{p_c}$$

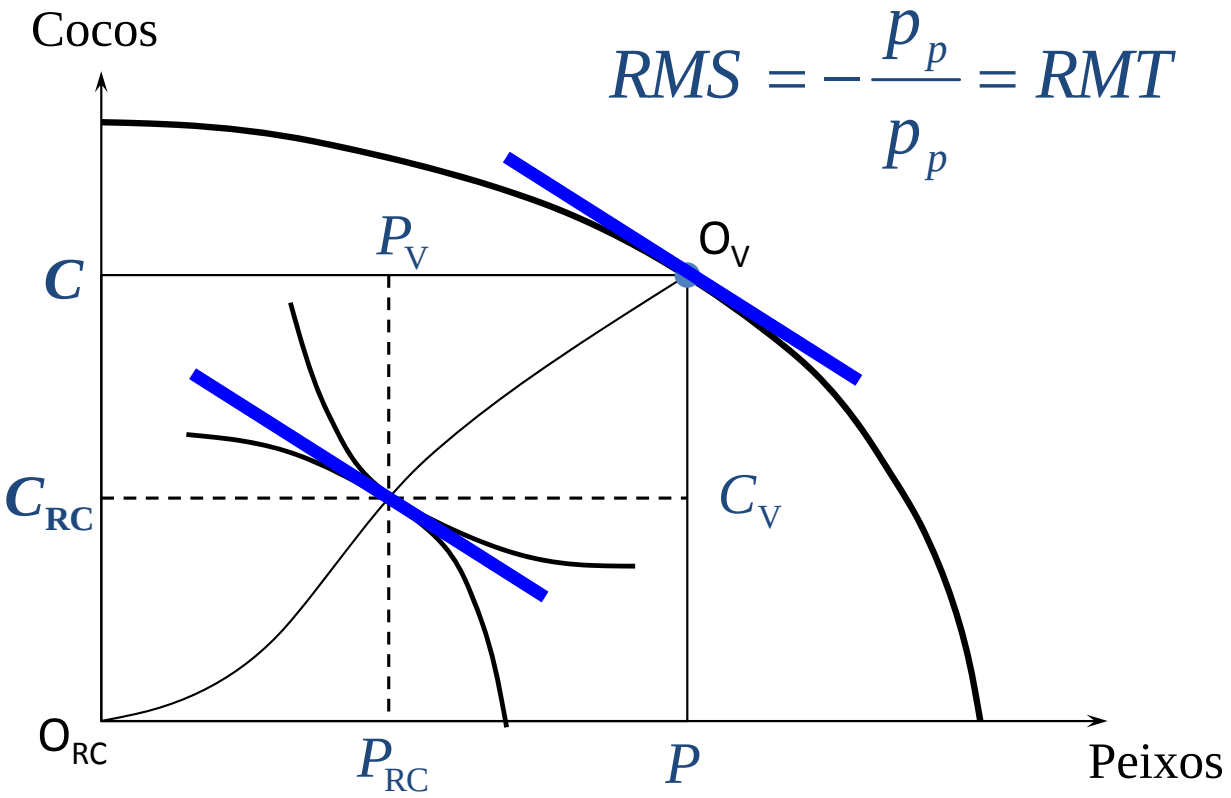


Efficiency of the competitive equilibrium

- When consumers optimize, they choose the basket such that

$$MRS_i = -\frac{p_f}{p_c},\ i = RC, F$$

- Hence, the decentralized allocation of resources in a competitive equilibrium is efficient!



THE END